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# How to Calculate Dimensionless Numbers in Fluid Mechanics and Their Applications in Chemical Engineering Processes: Reynolds, Mach, Froude, Euler, Power, Stokes, Weber, Capillary, and Cavitation Numbers

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## ABSTRACT

This paper explains how to calculate and interpret dimensionless numbers in fluid mechanics for chemical engineering applications, focusing on the Reynolds, Mach, Froude, Euler, Power, Stokes, Weber, Capillary, and Cavitation numbers. The first part presents each dimensionless number individually based on its definition, formula, required variables, threshold values, physical meaning, calculation example, interpretation, and handling strategy. The second part presents integrated case examples to show how several dimensionless numbers can be used together to analyze more complex chemical engineering systems. The examples include pipe flow, gas flow, heat exchangers, slurry pipelines, pumps, valves, nozzles, mixing vessels, wastewater aeration basins, aerated fermentation bioreactors, spray drying systems, cyclones, and particle-laden flows. Through these examples, this paper aims to help readers connect dimensionless-number calculations with process behavior, engineering interpretation, and practical decision-making in chemical engineering.

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## 1. INTRODUCTION

Fluid mechanics is one of the fundamental subjects in chemical engineering because many industrial processes involve the flow of liquids, gases, slurries, droplets, bubbles, and multiphase systems (Nandiyanto *et al.*, 2025). Fluid-flow phenomena are commonly found in pipelines, pumps, valves, heat exchangers, reactors, mixing vessels, slurry transport systems, wastewater aeration basins, aerated bioreactors, spray drying systems, nozzles, cyclones, and separation equipment. In these systems, flow behavior is affected by physical properties and operating variables such as density, viscosity, velocity, pressure, pipe diameter, impeller speed, particle size, droplet size, gravity, surface tension, and vapor pressure. Therefore, a clear understanding of fluid mechanics is required for process design, operation, troubleshooting, scale-up, and equipment safety. The principles of fluid mechanics have been widely discussed in classical and chemical-engineering-oriented textbooks, including topics such as fluid properties, momentum transfer, pipe flow, turbulence, dimensional analysis, pumps, valves, particulate systems, mixing, heat transfer equipment, and separation processes (Darby and Chhabra, 2017; Deen, 2016; Kay and Nedderman, 1974; Mory, 2011; Raju, 2011; Schetz and Fuhs, 1999). Previous studies have also explained the importance of fluid mechanics in chemical engineering from the viewpoints of principles, applications, research trends, and bibliometric insights (Nandiyanto *et al.*, 2025). For simplifying fluid mechanics problems, dimensionless numbers are useful tools because they combine several physical variables into meaningful ratios. These numbers help identify dominant physical effects, compare different systems, interpret flow regimes, evaluate pressure loss, estimate mixing power, analyze particle response, assess droplet or bubble deformation, and predict cavitation risk. In chemical engineering practice, dimensionless numbers are not only theoretical parameters but also practical indicators for decision-making in process design and industrial operation. However, students and engineers often still face difficulties in connecting formulas with practical interpretation, especially when several physical effects occur simultaneously in real process equipment. In fact, understanding engineering concepts is also important in education, especially when students are required to connect theoretical formulas with practical applications. (Susilawati, 2024; Nandiyanto *et al.*, 2026).

This paper aims to explain how selected dimensionless numbers in fluid mechanics can be calculated, interpreted, and applied in chemical engineering processes. The discussion focuses on the Reynolds (*Re*), Mach (*Ma*), Froude (*Fr*), Euler (*Eu*), Power (*Np*), Stokes (*Stk*), Weber (*We*), Capillary (*Ca*), and Cavitation ($\sigma c$) numbers because these numbers represent important physical effects commonly found in process systems, such as inertia, viscosity, pressure drop, gravity, compressibility, mixing power, particle response, surface tension, interfacial deformation, and cavitation. The paper mainly consists of two parts:

(i) Each dimensionless number is discussed individually based on its definition, formula, required variables, threshold values, physical meaning, calculation example, engineering interpretation, and handling strategy. This individual discussion is intended to help readers understand what each number means and how its value can be used to describe a specific fluid-flow phenomenon.

(ii) Integrated case examples are presented to show how several dimensionless numbers can be used together in one chemical engineering system. These examples are important because real process equipment usually involves more than one physical phenomenon at the same time. For example, a spray drying system may involve gas flow, droplet motion, particle transport, pressure drop, and interfacial deformation, while a mixing vessel may involve turbulence, vortex formation, mixing power, particle suspension, and droplet breakup.

This paper not only presents calculation procedures but also connects the calculated values with process behavior, equipment operation, troubleshooting, and practical engineering decisions. This approach is expected to help students, researchers, and practicing engineers understand fluid mechanics more practically by linking formulas, numerical results, physical interpretation, and real chemical engineering applications.

## 2. METHODS

This paper uses a conceptual and calculation-based approach to explain the use of dimensionless numbers in fluid mechanics for chemical engineering applications. The selected dimensionless numbers are the *Re*, *Ma*, *Fr*, *Eu*, *Np*, *Stk*, *We*, *Ca*, and σc numbers. These numbers were selected because they represent important physical effects in chemical engineering fluid-flow systems, including inertia, viscosity, pressure drop, gravity, compressibility, mixing power, particle response, surface tension, interfacial deformation, and cavitation. The discussion is arranged in two main parts.

(i) The first part explains each dimensionless number individually. Each number is presented based on its definition, formula, required variables, threshold values, or general interpretation ranges, physical meaning, calculation example, engineering interpretation, and handling strategy. This part is intended to show how each calculated value can be used to describe a specific fluid-flow phenomenon.

(ii) The second part presents integrated case examples in chemical engineering equipment and process systems. In this part, several dimensionless numbers are applied together to analyze systems that involve more than one physical phenomenon. The examples include pipelines, pumps, valves, heat exchangers, mixing vessels, slurry transport systems, wastewater aeration basins, aerated fermentation bioreactors, spray drying systems, nozzles, cyclones, and particle-laden flows. For each case, the relevant operating parameters are identified, the appropriate dimensionless numbers are selected and calculated, and the results are interpreted to explain process behavior and support practical engineering decisions.

## 3. RESULTS AND DISCUSSION

### 3.1. Concept of Dimensionless Numbers in Fluid Mechanics

Dimensionless numbers are used to simplify fluid mechanics analysis by converting physical variables into ratios without units. In chemical engineering, these numbers help engineers identify the dominant force or phenomenon in a process system. Before calculating dimensionless numbers, the main physical parameters must be understood. These parameters are summarized in **Table 1**. Using the physical parameters in **Table 1**, several dimensionless numbers can be calculated. The relationship between the selected dimensionless numbers and chemical engineering applications is illustrated in **Figure 1**. The selected dimensionless numbers, formulas, meanings, and application examples are shown in **Table 2**. Each dimensionless number gives a different engineering interpretation. Dimensionless numbers are not only mathematical values. They are practical tools for interpreting fluid-flow behavior and making engineering decisions in chemical engineering processes.

**Table 1.** Common physical parameters used in fluid mechanics calculations.

| NO | PARAMETER | MEANING | EXAMPLE |
|---|---|---|---|
| 1 | Density ($\rho$) | Mass per unit volume of fluid (general) | $\rho$ for Water = 1000 kg/m³ |
| 2 | Gas density ($\rho_g$) | Mass per unit volume of gas | $\rho$ for Air = 1.2 kg/m³ |
| 3 | Liquid density ($\rho_l$) | Mass per unit volume of liquid | $\rho$ for Water = 1000 kg/m³ |
| 4 | Particle density ($\rho_p$) | Mass per unit volume of a solid particle | $\rho$ for sand particle = 2650 kg/m³ |
| 5 | Mixed-fluid density ($\rho_{mix}$) | Density of mixed fluid or mixture in a vessel | $\rho$ for starch slurry = 1100 kg/m³ |
| 6 | Dynamic viscosity ($\mu$) | Resistance of fluid to flow | $\mu$ for Water = $1.0 \times 10^{-3}$ Pa.s |
| 7 | Kinematic viscosity ($\upsilon$) | Dynamic viscosity divided by density | $\upsilon$ for Water = $1.0 \times 10^{-6}$ m²/s |
| 8 | Velocity ($v$) | Average linear fluid velocity | Pipe flow = 2 m/s |
| 9 | Gas velocity ($vg$) | Average gas flow velocity | Velocity of drying air: 20 m/s |
| 10 | Liquid velocity ($vl$) | Average liquid flow velocity | Velocity of cooling water: 1.5 m/s |
| 11 | Pump or nozzle velocity ($vp$) | Liquid velocity in a pump, valve, or nozzle | Velocity in pump line = 8 m/s |
| 12 | Volumetric flow rate ($Q$) | Volume of fluid flowing per unit time | Air flow = 3 m³/min |
| 13 | Cross-sectional area ($A$) | Flow area of pipe or duct | Pipe area cross-section= 0.785 m² |
| 14 | Pipe diameter ($Dp$) | Internal diameter of pipe | Pipe diameter: 0.05 m |
| 15 | Impeller diameter ($D_{imp}$) | Diameter of mixing impeller | $D_{imp}$ = 0.30 m = 30 cm |
| 16 | Droplet or bubble diameter ($d_b$) | Diameter of droplet or bubble | $d_b$ = 0.002 m |
| 17 | Particle diameter ($d_p$) | Diameter of a solid particle | $d_p$ = 100 µm = $1.0 \times 10^{-4}$ m |
| 18 | Characteristic length ($L$) | Representative system length | Tank length = 1.2 m |
| 19 | Pressure drop ($\Delta P$) | Pressure loss in the pipe or equipment | $\Delta P$ = 1500 Pa |
| 20 | Local pressure ($P$) | Absolute pressure at a selected point | Pressure in pump inlet = 200 kPa |
| 21 | Vapor pressure ($Pv$) | Pressure at which a liquid vaporizes | Water vapor pressure = 12,000 Pa |
| 22 | Gravity ($g$) | Gravitational acceleration | $g$ = 9.81 m/s² |
| 23 | Surface tension ($\sigma$) | Interfacial force between fluids | $\sigma$ for Water-air: 0.072 N/m |
| 24 | Speed of sound ($c$) | Sound velocity in gas on average | $c_{Air}$ average = 346 m/s |
| 25 | Speed of sound at a specific temperature ($c_T$) | Sound velocity in a gas at a specific temperature | $c_{Air}$ at 500°C = 557 m/s |
| 26 | Heat capacity ratio ($\gamma$) | Ratio of $Cp$ to $Cv$ for a gas | $\gamma$ Air = 1.4 |
| 27 | Specific gas constant ($R_{spec}$) | Gas constant per unit mass | $R_{spec}$ Air = 287 J/kg.K |
| 28 | Absolute temperature ($T$) | Gas temperature in Kelvin | T at 25°C = 298 K |
| 29 | Power input ($P_W$) | Energy input per unit time | Power for mixer: 120 W |
| 30 | Rotational speed ($N$) | Impeller rotation per second | $N$ = 5 1/s |
| 31 | Particle response time ($\tau_p$) | Time required for a particle to respond to fluid motion | $1.47 \times 10^{-3}$ s |
| 32 | Fluid characteristic time ($\tau f$) | Characteristic time scale of fluid motion | $5.0 \times 10^{-3}$ s |
| 33 | Fluid density ($\rho f$) | Density of flowing fluid or continuous fluid phase | Air: 1.2 kg/m³; Water: 1000 kg/m³ |
| 34 | Characteristic velocity ($vc$) | Representative velocity of the analyzed flow phenomenon | Gas velocity in spray dryer: 20 m/s |
| 35 | Characteristic length ($Lc$) | Representative length scale of the analyzed flow phenomenon | Spray region length: 0.50 m |
| 36 | Fluid dynamic viscosity ($\mu f$) | Dynamic viscosity of the continuous fluid around the particle | Air: $2.0 \times 10^{-5}$ Pa.s; Water: $1.0 \times 10^{-3}$ Pa.s |

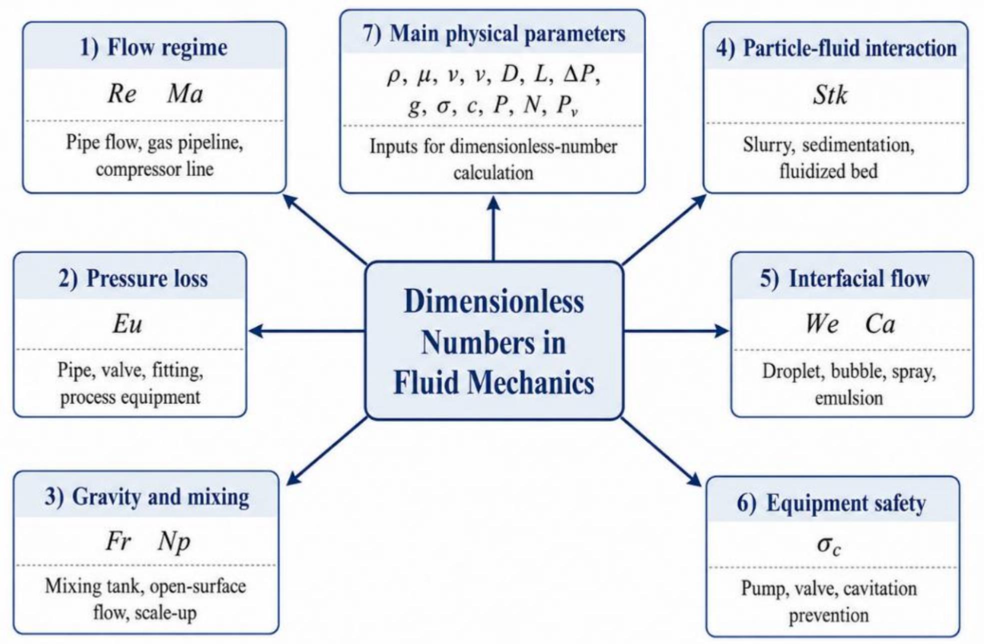


**Figure 1.** Conceptual map of dimensionless numbers and chemical engineering applications.

**Table 2.** Selected dimensionless numbers, formulas, meanings, and applications.

| NO | DIMENSIONLESS NUMBER | MAIN TOPIC | PHYSICAL RATIO | MAIN QUESTION ANSWERED | EXAMPLE CASE |
|---|---|---|---|---|---|
| 1 | Reynolds: $Re = \frac{\rho v D_p}{\mu}$ | Flow regime | Inertial force / viscous force | Is the flow laminar or turbulent? | Gas or liquid flow in a pipe |
| 2 | Mach: $Ma = \frac{v_g}{c_T}$ | Compressible gas flow | Flow velocity / speed of sound | Is gas compressibility important? | Gas pipeline |
| 3 | Froude: $Fr = \frac{v}{\sqrt{gL}}$ | Free-surface and mixing flow | Inertial force / gravitational force | Is gravity important? | Mixing tank |
| 4 | Euler: $Eu = \frac{\Delta P}{\rho v^2}$ | Pressure drop | Pressure force / inertial force | Is pressure loss significant? | Pipe, valve, or fitting |
| 5 | Power: $Np = \frac{P_w}{\rho_{mix} N^3 D_{imp}^5}$ | Mixing power | Power input / inertial mixing scale | How much power is required? | Agitator system |
| 6 | Stokes: $Stk = \frac{\tau_p}{\tau_f}$ | Particle-fluid interaction | Particle response time / fluid time scale | Do particles follow the fluid flow? | Slurry flow |
| 7 | Weber: General: $We = \frac{\rho_f v_c^2 L_c}{\sigma}$ Droplet: $We = \frac{\rho_f v_c^2 d_b}{\sigma}$ | Interfacial flow | Inertial force / surface tension force | Will droplets or bubbles deform? | Spray or bubble formation |
| 8 | Capillary: $Ca = \frac{\mu_f v_c}{\sigma}$ | Interfacial flow | Viscous force / surface tension force | Does viscosity deform the interface? | Emulsion or coating |
| 9 | Cavitation: $\sigma_c = \frac{P_{loc} - P_v}{0.5 \rho_l v_p^2}$ | Equipment safety | Pressure margin / dynamic pressure | Is cavitation likely? | Pump or valve |

### 3.1.1. Physical, Operating, and Characteristic Parameters Used in Dimensionless-Number Calculations

Before calculating dimensionless numbers, the required physical, operating, and characteristic parameters must be selected correctly, since they depend strongly on fluid properties, operating conditions, equipment geometry, and the characteristic size.

Several physical properties depend on the type of material and operating temperature:

(i) Density (ρ) (**Table 3**) affects inertial effects and is used in several dimensionless numbers, such as *Re*, *Eu*, *We*, *Np*, *Stk*, and σc numbers.

(ii) Dynamic viscosity (μ) (**Table 4**) represents the resistance of a fluid to flow and is important in *Re*, *Ca*, and *Stk* number calculations.

(iii) Surface tension (σ) (**Table 5**) represents the interfacial force between phases and is used in *We* and *Ca* number calculations.

(iv) Vapor pressure (*Pv*) is used in σc number calculations and depends strongly on liquid type and temperature. Since vapor pressure generally increases with temperature, hot liquids have a smaller pressure margin above vapor pressure and are more prone to cavitation.

**Table 3.** Approximate density, $\rho$ ($kg/m^3$), at different temperatures.

| NO | MATERIAL/ FLUID | $\rho$ ($kg/m^3$) | | | |
|---|---|---|---|---|---|
| | | 25°C | 50°C | 100°C | 200°C |
| 1 | Water | 997 | 988 | 958 | 865 |
| 2 | Wastewater, dilute | 995-1010 | 985-1000 | 950-970 | N/A |
| 3 | Syrup, sugar solution | 1200-1400 | 1180-1360 | 1120-1300 | N/A |
| 4 | Slurry, water-solid mixture | 1100-1500 | 1080-1480 | 1040-1400 | N/A |
| 5 | Methanol | 787 | 765 | 710 | 550-600 |
| 6 | Ethanol | 785 | 763 | 716 | 560-620 |
| 7 | Isopropanol | 781 | 760 | 700 | 520-600 |
| 8 | n-Propanol | 799 | 780 | 730 | 600-650 |
| 9 | n-Butanol | 810 | 790 | 740 | 620-680 |
| 10 | Acetone | 785 | 755 | 690 | 450-550 |
| 11 | Methyl ethyl ketone | 800 | 775 | 720 | 520-600 |
| 12 | Acetonitrile | 776 | 755 | 700 | 520-600 |
| 13 | Ethyl acetate | 894 | 870 | 800 | 600-700 |
| 14 | Diethyl ether | 707 | 670 | N/A | N/A |
| 15 | Dichloromethane | 1320 | 1270 | 1150-1200 | N/A |
| 16 | Chloroform | 1480 | 1430 | 1300-1350 | 900-1050 |
| 17 | Carbon tetrachloride | 1580 | 1530 | 1400-1450 | 1050-1150 |
| 18 | Benzene | 874 | 850 | 780 | 600-700 |
| 19 | Toluene | 862 | 840 | 780 | 650-720 |
| 20 | o-Xylene | 875 | 855 | 810 | 700-760 |
| 21 | n-Hexane | 655 | 630 | 560-600 | 350-450 |
| 22 | n-Heptane | 680 | 655 | 600 | 450-550 |
| 23 | n-Octane | 698 | 675 | 625 | 520-600 |
| 24 | Cyclohexane | 774 | 745 | 680 | 500-600 |
| 25 | Acetic acid | 1044 | 1020 | 940 | 760-850 |
| 26 | Formic acid | 1210 | 1180 | 1080 | 850-950 |
| 27 | Ethylene glycol | 1110 | 1085 | 1040 | 930-980 |
| 28 | Propylene glycol | 1035 | 1010 | 965 | 850-920 |
| 29 | Glycerol | 1260 | 1235 | 1190 | 1080-1150 |
| 30 | Dimethyl sulfoxide, DMSO | 1095 | 1070 | 1020 | 880-950 |

**Table 4.** Approximate dynamic viscosity, μ, at different temperatures.

| NO | MATERIAL/ FLUID | μ (mPa.s) | | | |
|---|---|---|---|---|---|
| | | 25°C | 50°C | 100°C | 200°C |
| 1 | Water | 0.89 | 0.55 | 0.28 | 0.13 |
| 2 | Wastewater, dilute | 0.9-1.5 | 0.6-1.0 | 0.3-0.6 | N/A |
| 3 | Syrup, sugar solution | 100-5000 | 20-1000 | 5-200 | N/A |
| 4 | Slurry, water-solid mixture | 2-200 | 1-100 | 0.5-50 | N/A |
| 5 | Methanol | 0.54 | 0.38 | 0.22 | 0.10-0.15 |
| 6 | Ethanol | 1.07 | 0.70 | 0.35 | 0.15-0.20 |
| 7 | Isopropanol | 2.0 | 1.2 | 0.45 | 0.15-0.25 |
| 8 | n-Propanol | 1.9 | 1.1 | 0.45 | 0.18-0.25 |
| 9 | n-Butanol | 2.6 | 1.5 | 0.60 | 0.25-0.35 |
| 10 | Acetone | 0.31 | 0.24 | 0.15 | 0.07-0.12 |
| 11 | Methyl ethyl ketone | 0.40 | 0.30 | 0.18 | 0.08-0.13 |
| 12 | Acetonitrile | 0.34 | 0.26 | 0.16 | 0.08-0.12 |
| 13 | Ethyl acetate | 0.43 | 0.32 | 0.18 | 0.08-0.13 |
| 14 | Diethyl ether | 0.22 | 0.16 | N/A | N/A |
| 15 | Dichloromethane | 0.41 | 0.31 | 0.18 | N/A |
| 16 | Chloroform | 0.56 | 0.42 | 0.25 | 0.10-0.15 |
| 17 | Carbon tetrachloride | 0.90 | 0.65 | 0.36 | 0.15-0.22 |
| 18 | Benzene | 0.60 | 0.43 | 0.25 | 0.10-0.16 |
| 19 | Toluene | 0.56 | 0.42 | 0.28 | 0.13-0.18 |
| 20 | o-Xylene | 0.75 | 0.55 | 0.35 | 0.16-0.23 |
| 21 | n-Hexane | 0.30 | 0.23 | 0.14 | 0.06-0.10 |
| 22 | n-Heptane | 0.39 | 0.29 | 0.18 | 0.08-0.12 |
| 23 | n-Octane | 0.51 | 0.38 | 0.24 | 0.10-0.15 |
| 24 | Cyclohexane | 0.90 | 0.62 | 0.34 | 0.13-0.20 |
| 25 | Acetic acid | 1.1 | 0.75 | 0.35 | 0.15-0.25 |
| 26 | Formic acid | 1.6 | 1.0 | 0.45 | 0.18-0.30 |
| 27 | Ethylene glycol | 16 | 6-8 | 2-3 | 0.7-1.0 |
| 28 | Propylene glycol | 40-50 | 10-15 | 3-5 | 1-2 |
| 29 | Glycerol | 900-1200 | 120-180 | 15-25 | 2-5 |
| 30 | Dimethyl sulfoxide, DMSO | 2.0 | 1.3 | 0.6 | 0.25-0.40 |

In addition to physical properties, operating parameters are also required. These parameters include velocity, volumetric flow rate, pressure drop, local pressure, temperature, power input, and impeller speed.

(i) Velocity is used in *Re*, *Ma*, *Fr*, *Eu*, We, *Ca*, *Stk*, and σc number calculations.

(ii) Pressure drop ($\Delta P$) is used in the *Eu* number to evaluate hydraulic resistance or energy loss. Local pressure ($P_{loc}$) is used in the σc number to evaluate whether the liquid pressure is close to the vapor pressure.

(iii) Power input (*Pw*), impeller speed (*N*), and impeller diameter ($D_{imp}$) are used in the *Np* to evaluate mixing power requirements.

The selection of characteristic size is also important.

(i) For pipe-flow analysis, the internal pipe diameter (*Dp*) is commonly used as the characteristic length. For mixing systems, the impeller diameter ($D_{imp}$), or tank diameter ($D_T$) may be used depending on the analyzed phenomenon.

(ii) For systems involving dispersed phases, the characteristic size should represent the object being analyzed. The droplet diameter (*dd*) is used when the system involves liquid droplets, such as in spray drying, atomization, nozzle spray, mist formation, or emulsions. The bubble diameter (*db*) is used when the system involves gas bubbles, such as in wastewater aeration, aerated bioreactors, spargers, diffusers, and bubble columns. The particle diameter (*dp*) is used when the system involves solid particles, such as in slurry flow, cyclone separation, sedimentation, crystallization, powder flow, and particle-laden flow. When the analyzed system does not specifically focus on droplets, bubbles, or particles, a general characteristic length (*Lc*) can be used, such as pipe diameter, hydraulic diameter, tank diameter, chamber diameter, nozzle diameter, or spray-zone length.

**Table 5.** Approximate surface tension, σ, at different temperatures

| NO | MATERIAL / FLUID | σ (mN/m) | | | |
|---|---|---|---|---|---|
| | | 25°C | 50°C | 100°C | 200°C |
| 1 | Water | 72 | 68 | 59 | 38-45 |
| 2 | Wastewater, dilute | 60-72 | 55-68 | 45-60 | N/A |
| 3 | Syrup, sugar solution | 65-75 | 60-70 | 50-65 | N/A |
| 4 | Slurry, water-solid mixture | 60-75 | 55-70 | 45-65 | N/A |
| 5 | Methanol | 22 | 19 | 12-15 | 2-8 |
| 6 | Ethanol | 22 | 19 | 13-16 | 3-8 |
| 7 | Isopropanol | 21 | 18 | 12-15 | 2-7 |
| 8 | n-Propanol | 23 | 20 | 14-17 | 4-9 |
| 9 | n-Butanol | 24-25 | 21-22 | 15-18 | 5-10 |
| 10 | Acetone | 23 | 20 | 12-15 | 1-5 |
| 11 | Methyl ethyl ketone | 24 | 21 | 14-17 | 3-8 |
| 12 | Acetonitrile | 29 | 25 | 18-21 | 6-12 |
| 13 | Ethyl acetate | 24 | 21 | 14-17 | 3-8 |
| 14 | Diethyl ether | 16-17 | 12-14 | N/A | N/A |
| 15 | Dichloromethane | 27 | 22 | 12-16 | N/A |
| 16 | Chloroform | 27 | 23 | 16-19 | 5-10 |
| 17 | Carbon tetrachloride | 26-27 | 23 | 17-20 | 7-12 |
| 18 | Benzene | 28 | 24 | 17-20 | 6-12 |
| 19 | Toluene | 28 | 25 | 19-22 | 10-15 |
| 20 | o-Xylene | 30 | 27 | 21-24 | 12-18 |
| 21 | n-Hexane | 18 | 15 | 8-12 | 1-4 |
| 22 | n-Heptane | 20 | 17 | 11-14 | 3-7 |
| 23 | n-Octane | 21 | 19 | 13-16 | 5-9 |
| 24 | Cyclohexane | 25 | 21 | 14-17 | 3-8 |
| 25 | Acetic acid | 27 | 24 | 17-21 | 8-14 |
| 26 | Formic acid | 37 | 33 | 25-28 | 10-18 |
| 27 | Ethylene glycol | 47-48 | 44 | 36-40 | 22-28 |
| 28 | Propylene glycol | 36 | 32 | 25-30 | 15-22 |
| 29 | Glycerol | 63 | 58 | 50-55 | 35-45 |
| 30 | Dimethyl sulfoxide, DMSO | 43 | 39 | 30-35 | 18-25 |

The use of droplet diameter is case-specific and should not be applied to all fluid-flow systems (**Table 6)**. For example, *dd* is suitable for *We* number calculations in spray drying because the deformation or breakup of liquid droplets is being analyzed. However, for wastewater aeration, the relevant dispersed object is a gas bubble; *db* should be used. For slurry transport or cyclone separation, the relevant dispersed object is a solid particle; *dp* should be used.

**Table 6.** Typical droplet, bubble, or particle diameter ranges produced in different equipment.

| NO | EQUIPMENT / PROCESS | PRODUCED DISPERSED PHASE | TYPICAL DIAMETER RANGE | NOTES |
|---|---|---|---|---|
| 1 | Two-fluid nozzle / pneumatic atomizer | Fine droplets | 1–100 µm | Common in lab spray drying and fine atomization |
| 2 | Pressure nozzle / hydraulic nozzle | Droplets | 10–400 µm | Depends strongly on pressure, orifice size, and viscosity |
| 3 | Rotary atomizer / centrifugal disk | Droplets | 10–500 µm | Common in industrial spray drying; controlled by disk speed and feed rate |
| 4 | Ultrasonic atomizer / nebulizer | Very fine droplets | 1–50 µm | Produces relatively fine droplets; droplet size depends on frequency and liquid properties |
| 5 | Spray dryer feed atomization | Droplets before drying | 5–500 µm | Depends on atomizer type and feed properties |
| 6 | Spray dryer dried powder | Dry particles | 1–200 µm | Final particle size is not always equal to initial droplet size |
| 7 | Air diffuser in wastewater aeration | Air bubbles | 0.5–5 mm | Fine-bubble diffusers produce smaller bubbles than coarse-bubble diffusers |
| 8 | Coarse-bubble diffuser | Air bubbles | 5–20 mm | Used when strong mixing is more important than high oxygen transfer efficiency |
| 9 | Bubble column / gas sparger | Gas bubbles | 1–10 mm | Affected by gas velocity, sparger hole size, liquid viscosity, and coalescence |
| 10 | Aerated stirred tank / bioreactor | Gas bubbles | 0.5–5 mm | Smaller bubbles can form at high agitation and suitable sparging |
| 11 | Mechanical mixing for emulsion | Liquid droplets | 1–1000 µm | Strongly affected by impeller speed, emulsifier, viscosity ratio, and mixing time |
| 12 | High-shear mixer / rotor-stator | Emulsion droplets | 0.5–100 µm | Used for finer emulsions and dispersions |
| 13 | Homogenizer / high-pressure homogenizer | Emulsion droplets | 0.1–10 µm | Produces very fine droplets under high pressure |
| 14 | Microfluidic droplet generator | Monodisperse droplets | 10–500 µm | Droplet size is controlled by channel geometry and flow-rate ratio |
| 15 | Inkjet / drop-on-demand nozzle | Droplets | 10–100 µm | Depends on nozzle diameter, waveform, viscosity, and surface tension |
| 16 | Agricultural spray nozzle | Droplets | 50–1000 µm | Finer droplets drift more easily; coarse droplets deposit faster |
| 17 | Cooling tower spray nozzle | Water droplets | 100–5000 µm | Large droplets are common to reduce drift |
| 18 | Scrubber spray nozzle | Liquid droplets | 100–1000 µm | Used for gas cleaning and mass transfer |
| 19 | Fire sprinkler / water mist | Water droplets | 50–5000 µm | Water mist systems produce much smaller droplets than ordinary sprinklers |
| 20 | Slurry pipeline | Solid particles | 10–1000 µm | Particle size depends on solid material and grinding/separation process |
| 21 | Wastewater flocs | Flocs / suspended particles | 10–1000 µm | Floc size changes with coagulation, shear, and settling |
| 22 | Cyclone separator | Dust particles | 1–100 µm | Separation efficiency depends strongly on particle size and density |
| 23 | Pneumatic conveying | Solid particles | 10–5000 µm | Depends on powder type and conveying mode |
| 24 | Crystallizer suspension | Crystals | 10–1000 µm | Depends on nucleation, growth, and breakage |
| 25 | Fluidized bed granulation | Granules | 100–3000 µm | Granule size depends on binder spray, drying, and agglomeration |

Typical values of these parameters can be obtained from property databases, handbooks, equipment specifications, experimental measurements, image analysis, particle size analysis, or literature data. For preliminary educational calculations, representative values or typical ranges can be used. However, for detailed design and accurate engineering analysis, the actual properties and characteristic sizes should be selected based on the real material, operating temperature, equipment geometry, and process condition. The values obtained must be validated from database such as NIST Chemistry WebBook, DIPPR, Perry's Handbook, Aspen, or REFPROP. N/A indicates that the material may not exist as a stable liquid at the stated temperature under normal conditions, may exceed its critical temperature, or may undergo thermal degradation. For complex fluids such as wastewater, syrup, and slurry, the values depend on composition, solid content, dissolved substances, and temperature; therefore, representative ranges are given.

### 3.1.2. Reynolds Number: Flow Regime and Inertial-Viscous Balance

Reynolds number, *Re*, is one of the most important dimensionless numbers in fluid mechanics. It is used to classify flow regimes and to determine whether the flow is laminar, transitional, or turbulent. For internal flow in a circular pipe, the common interpretation of *Re* is presented in **Table 7**.

**Table 7.** *Re* number interpretation for pipe flow.

| RANGE | FLOW REGIME | MEANING |
|---|---|---|
| *Re* < 2100 | Laminar | Viscous force dominates, and the flow is smooth |
| *Re* = 2100 - 4000 | Transitional | The flow condition is unstable |
| *Re* > 4000 | Turbulent | Inertial force dominates, and the flow disturbance is strong |

To show the practical use of the *Re* number, three industrial examples are presented in **Table 8**. These examples represent syrup flow in the food industry, cooling water flow in a heat exchanger, and slurry flow in a wastewater treatment plant.

**Table 8.** Input data for industrial Re number examples.

| CASE | INDUSTRIAL APPLICATION | $\rho$ (kg/m³) | *v* (m/s) | *D* (m) | $\mu$ (Pa.s) |
|---|---|---|---|---|---|
| 1 | Syrup flow in the food industry | 1300 | 0.10 | 0.04 | 0.500 |
| 2 | Cooling water in the heat exchanger | 1000 | 1.50 | 0.05 | 0.001 |
| 3 | Slurry flow in wastewater treatment | 1100 | 0.80 | 0.08 | 0.020 |

The *Re* is calculated using $Re = \frac{\rho v D}{\mu}$, and the calculation results, industrial interpretation, and engineering handling strategies are summarized in **Table 9**.

(i) For the first case, syrup is transferred from a storage tank to a filling machine through a pipe. The *Re* number is calculated as follows: *Re* = 1300 x 0.10 x 0.04 / 0.50 = 10.4. This condition is classified as laminar flow.

(ii) For the second case, cooling water is circulated through a pipe to a heat exchanger. The *Re* number is calculated as follows: *Re* = 1000 x 1.50 x 0.05 / 0.001 = 75,000. This condition is classified as turbulent flow.

(iii) For the third case, a slurry containing water and fine solid particles is transported through a pipe to a sedimentation unit. The *Re* number is calculated as follows: *Re* = 1100 x 0.80 x 0.08 / 0.02 = 3,520. This condition is classified as transitional flow.

Each *Re* number range requires a different engineering response. Laminar flow may be acceptable for smooth fluid transfer, but it is less effective for mixing. Turbulent flow is useful for heat transfer and mixing, but it can increase pressure drop, energy consumption, noise, and vibration. Transitional flow should be carefully controlled because it may cause unstable operation, especially in slurry transport systems. Therefore, the *Re* number can be used as a practical engineering tool for process design and troubleshooting. By calculating *Re* number, engineers can identify the flow regime, predict possible operational problems, and determine suitable handling strategies for industrial fluid-flow systems.

**Table 9.** Summary of *Re* number results and engineering handling strategies.

| CASE | INDUSTRIAL APPLICATION | *Re* VALUE | FLOW REGIME | INDUSTRIAL INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|
| 1 | Syrup flow in the food industry | 10.40 | Laminar | The syrup flows smoothly, but mixing inside the pipe is weak because of high viscosity. | Use a mixer before pipe flow if a uniform composition is required. Slightly increase the temperature to reduce viscosity, or adjust the pump velocity carefully. |
| 2 | Cooling water in the heat exchanger | 75,000 | Turbulent | The flow disturbance is strong, which is beneficial for heat transfer. However, pressure drop and pumping energy may increase. | Check pump capacity, pipe diameter, total pressure drops, fouling, fittings, and vibration. Reduce unnecessary bends and clean the pipe regularly. |
| 3 | Slurry flow in wastewater treatment | 3,520 | Transitional | The flow is unstable and may not be sufficient to keep solid particles fully suspended. | Increase flow velocity if particle settling occurs. Avoid dead zones, use abrasion-resistant pipe material, and select a suitable slurry pump. |

### 3.1.3. Mach Number: Gas Compressibility in Industrial Flow Systems

Mach number, *Ma*, is used to evaluate the importance of compressibility effects in gas flow. It is defined as the ratio between gas flow velocity and the speed of sound in the gas. In chemical engineering applications, Mach number is important for gas pipelines, compressed air systems, steam lines, nozzles, valves, spray dryers, and other high-velocity gas transport systems. The Mach number is calculated using $Ma = v / c$, where v is the gas flow velocity and *c* is the speed of sound in the gas. The gas velocity, *v*, represents how fast the gas moves through a pipe, nozzle, dryer, or other equipment. Meanwhile, the speed of sound, c, represents how fast pressure disturbances travel through the gas. The value of *c* is not constant for all gases. It depends on gas properties and temperature. Therefore, different gases may have different speeds of sound values even under similar operating conditions. In general, the speed of sound increases when the gas temperature increases. Approximate values of the speed of sound for different gases at several temperatures are presented in **Table 10**. The general interpretation of Mach number is presented in **Table 11**.

**Table 10.** Approximate speed of sound in different gases at different temperatures.

| NO | GAS | FORMULA | $c$ (m/s) | | | | COMMON INDUSTRIAL APPLICATION |
|---|---|---|---|---|---|---|---|
| | | | 20°C | 100°C | 200°C | 500°C | |
| 1 | Air | Air | 343 | 387 | 436 | 557 | Compressed air, ventilation, spray drying |
| 2 | Oxygen | $O_2$ | 326 | 368 | 414 | 529 | Combustion, oxidation |
| 3 | Nitrogen | $N_2$ | 349 | 394 | 443 | 567 | Inerting, purging, blanketing |
| 4 | Hydrogen | $H_2$ | 1306 | 1473 | 1659 | 2121 | Hydrogen pipeline, fuel cell |
| 5 | Helium | He | 1008 | 1137 | 1281 | 1637 | Leak detection, cryogenic system |
| 6 | Carbon dioxide | $CO_2$ | 268 | 302 | 340 | 435 | Carbonation, refrigeration |
| 7 | Carbon monoxide | CO | 349 | 394 | 443 | 567 | Syngas, metallurgical process |
| 8 | Methane | $CH_4$ | 446 | 503 | 567 | 724 | Natural gas, fuel gas |
| 9 | Ammonia | $NH_3$ | 433 | 489 | 550 | 703 | Refrigeration, fertilizer industry |
| 10 | Argon | Ar | 319 | 360 | 405 | 518 | Welding, inert shielding |
| 11 | Neon | Ne | 449 | 507 | 570 | 729 | Lighting, specialty gas |
| 12 | Krypton | Kr | 220 | 248 | 279 | 357 | Specialty lighting |
| 13 | Xenon | Xe | 176 | 199 | 224 | 286 | Specialty gas |
| 14 | Chlorine | $Cl_2$ | 214 | 241 | 272 | 348 | Chlor-alkali process |
| 15 | Sulfur dioxide | $SO_2$ | 222 | 250 | 282 | 361 | Sulfuric acid production |
| 16 | Nitrous oxide | $N_2O$ | 267 | 301 | 339 | 434 | Chemical process, medical gas |
| 17 | Nitric oxide | NO | 337 | 380 | 428 | 547 | Emission gas, nitric acid process |
| 18 | Hydrogen sulfide | $H_2S$ | 307 | 346 | 390 | 499 | Sour gas treatment |
| 19 | Hydrogen chloride | HCl | 307 | 346 | 390 | 499 | Acid gas system |
| 20 | Hydrogen fluoride | HF | 413 | 466 | 525 | 671 | Fluorination process |
| 21 | Ethylene | $C_2H_4$ | 328 | 370 | 417 | 533 | Polymerization |
| 22 | Ethane | $C_2H_6$ | 314 | 354 | 399 | 510 | Petrochemical feedstock |
| 23 | Acetylene | $C_2H_2$ | 343 | 387 | 436 | 557 | Welding, chemical synthesis |
| 24 | Propane | $C^3H_8$ | 250 | 282 | 318 | 406 | LPG, fuel gas |
| 25 | n-Butane | $C_4H_{10}$ | 214 | 241 | 272 | 348 | LPG, fuel gas |
| 26 | Isobutane | i-$C_4H_{10}$ | 214 | 241 | 272 | 348 | Refrigerant, LPG |
| 27 | Propylene | $C_3H_6$ | 258 | 291 | 328 | 419 | Petrochemical process |
| 28 | Vinyl chloride | $C_2H_3Cl$ | 213 | 240 | 271 | 346 | PVC production |
| 29 | Methyl chloride | $CH_3Cl$ | 241 | 272 | 306 | 391 | Chemical intermediate |
| 30 | Fluorine | $F_2$ | 295 | 333 | 375 | 479 | Fluorination process |

Note: The speed of sound can be estimated: $c = \sqrt{\gamma R_{specific} T}$, where $\gamma$ is the heat capacity ratio, $R_{specific}$ is the specific gas constant, and $T$ is the absolute temperature in Kelvin. As a function of time, $c_T = c_{20}\sqrt{\frac{T_K}{293.15}}$, where $c_T$ is the estimated speed of sound at a selected temperature, $c_{20}$ is the speed of sound at 20°C, and $T_K$ is the gas temperature in K. In aerosol systems such as spray drying, *c* generally refers to the continuous gas phase, such as hot air or drying gas, rather than the liquid droplets. Therefore, *c* should be selected based on the temperature and composition of the gas phase.

**Table 11.** General interpretation of the *Ma* number.

| *Ma* NUMBER RANGE | FLOW CONDITION | GENERAL MEANING |
|---|---|---|
| *Ma* < 0.3 | Nearly incompressible gas flow | Almost no change (Density change is usually small) |
| 0.3 ≤ *Ma* < 1 | Compressible subsonic flow | The compressibility effect becomes important |
| *Ma* = 1 | Sonic flow | Gas velocity reaches the speed of sound |
| *Ma* > 1 | Supersonic flow | Shock and strong compressibility effects may occur |

To show the practical use of the *Ma* number, three industrial examples are presented in **Table 12**. The calculation results, industrial interpretation, and engineering handling strategies are summarized in **Table 13**. These examples represent compressed air flow, natural gas transport, hot-air flow in a spray drying system, and the nozzle process:

(i) For the first case, compressed air flows through a utility pipeline in an industrial plant. The *Ma* number is calculated as follows: *Ma* = 20 / 346 = 0.058. This condition is classified as nearly incompressible gas flow.

(ii) For the second case, hot air flows inside a spray dryer. The *Ma* number is calculated as follows: *Ma* = 50 / 412 = 0.121. This condition is classified as nearly incompressible gas flow.

(iii) For the third case, natural gas flows through a transmission pipeline at relatively high velocity. The *Ma* number is calculated as follows: *Ma* = 120 / 400 = 0.300. This condition is at the beginning of the compressible subsonic flow region.

(iv) For the fourth case, gas flows through a nozzle throat under choking conditions: Ma = 346 / 346 = 1.000. This condition is classified as sonic flow.

(v) For the fifth case, a high-pressure gas jet leaves a nozzle at very high velocity: Ma = 600 / 346 = 1.734. This condition is classified as supersonic flow.

**Table 12.** Input data for industrial *Ma* number examples.

| CASE | INDUSTRIAL APPLICATION | GAS PHASE | *v* (m/s) | *c* (m/s) |
|---|---|---|---|---|
| 1 | Compressed air flow in a plant utility line | Air at room temperature | 20 | 346 |
| 2 | Hot-air flow in a spray dryer | Hot air at about 150°C | 50 | 412 |
| 3 | Natural gas flow in a transmission pipeline | Methane-rich natural gas | 120 | 400 |
| 4 | Gas flow at the nozzle throat | Air or steam near choking condition | 346 | 346 |
| 5 | High-speed gas jet from a nozzle | High-pressure gas jet | 600 | 346 |

The examples represent the main *Ma* number regions: nearly incompressible flow, compressible subsonic flow, sonic flow, and supersonic flow. *Ma* number is useful for determining whether gas compressibility should be considered in engineering calculations. When *Ma* is lower than 0.3, the gas flow can generally be treated as nearly incompressible because the density change is relatively small. However, when *Ma* approaches or exceeds 0.3, compressibility effects become more significant and should be included in the analysis.

In industrial systems, low-*Ma* flow is usually easier to analyze and handle because pressure, density, and velocity changes are relatively limited. In contrast, higher-*Ma* flow requires more careful design because gas properties and flow behavior may change significantly along the pipeline, nozzle, or equipment. In spray drying, the *Ma* number is useful for evaluating whether the drying gas or atomizing gas velocity is high enough to require compressibility correction. Sonic and supersonic conditions may occur in nozzles, high-pressure gas release, steam jets, gas ejectors, and safety relief systems. Therefore, *Ma* number can be used to select an appropriate gas-flow analysis method, check pipeline and nozzle design, and prevent operational problems such as excessive pressure drop, noise, vibration, shock formation, and choked flow.

**Table 13.** Summary of *Ma* number results and engineering handling strategies.

| CASE | INDUSTRIAL APPLICATION | Ma VALUE | FLOW CONDITION | INDUSTRIAL INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|
| 1 | Compressed air flow in a plant utility line | 0.058 | Nearly incompressible gas flow | The gas velocity is low compared with the speed of sound. Density change is small. | Ordinary pipe-flow analysis can be used. Check pressure drop and compressor capacity, but compressibility correction is usually not critical. |
| 2 | Hot-air flow in a spray dryer | 0.121 | Nearly incompressible gas flow | The drying gas velocity is still low compared with the speed of sound at the operating temperature. The compressibility effect is small. | Use hot-air properties at the actual drying temperature. Check gas velocity, drying-air distribution, pressure drop, nozzle condition, and energy consumption. |
| 3 | Natural gas flow in a transmission pipeline | 0.300 | Beginning of compressible subsonic flow | Compressibility starts to become important. Gas density and velocity may change along the pipeline. | Use compressible-flow calculation or correction. Check pressure profile, pipe diameter, gas velocity, and allowable pressure drop. |
| 4 | Gas flow at the nozzle throat | 1.000 | Sonic flow | Flow reaches the speed of sound. Choked flow may occur, and the mass flow rate may become limited. | Check nozzle throat area, upstream pressure, downstream pressure, and choking condition. Redesign the nozzle if the flow limitation is not acceptable. |
| 5 | High-speed gas jet from a nozzle | 1.734 | Supersonic flow | High-speed gas jet from a nozzle | Shock waves, strong noise, vibration, and large pressure changes may occur. |

### 3.1.4. Froude Number: Gravity Effect in Fluid Flow and Mixing

Froude number, *Fr*, is used to compare the inertial force with the gravitational force in a fluid-flow system. In chemical engineering applications, this number is important for systems involving free-surface flow, open-channel flow, mixing tanks, vortex formation, and scale-up of stirred vessels. The general interpretation of *Fr* is presented in **Table 14**.

**Table 14.** General interpretation of *Fr* number.

| ***Fr* RANGE** | **FLOW CONDITION** | **GENERAL MEANING** |
|---|---|---|
| *Fr* < 1 | Gravity-dominated flow | Gravitational force is stronger than inertial force |
| *Fr* = 1 | Critical condition | Inertial and gravitational effects are balanced |
| *Fr* > 1 | Inertia-dominated flow | Inertial force is stronger than gravitational force |

To show the practical use of *Fr* number, three industrial examples are presented in **Table 15**. The calculation results, industrial interpretation, and engineering handling strategies are summarized in **Table 16**. These examples represent liquid mixing in a stirred tank, wastewater flow in an open channel, and high-speed flow in a mixing tank using $Fr = \frac{v}{\sqrt{gL}}$:

(i) For the first case, wastewater flows through an open channel, resulting in *Fr* = 0.361. This condition is classified as gravity-dominated flow.

(ii) For the second case, a low-velocity spray region is considered in a spray dryer. This simplified example is used to evaluate whether the gas or spray motion is strongly affected by gravity, resulting in *Fr* = 0.583. This condition is also classified as gravity-dominated flow.

(iii) For the third case, a critical spray region is considered in a spray dryer. This case is added to show the condition where inertial and gravitational effects are nearly balanced in spray drying with *Fr* = 1. This condition is classified as critical flow. In spray drying, this means that droplet movement is influenced by both gas momentum and gravity. Small changes in drying-air velocity, droplet size, or chamber geometry may change the droplet trajectory and residence behavior.

(iv) For the fourth case, the flow is at the critical condition where inertial and gravitational effects are balanced, resulting in *Fr* = 1. This condition is classified as critical flow.

(v) For the fifth case, liquid flows at high speed in a mixing vessel, resulting in *Fr* = 1.606, classified as an inertia-dominated flow.

(vi) For the sixth case, high-velocity drying air flows near an atomizer in a spray dryer, resulting in *Fr* = 3.831. This condition is classified as inertia-dominated flow.

(vii) For the seventh case, very high-velocity atomizing gas is used in a spray dryer. This case is added to show a stronger inertia-dominated condition in spray drying, reaching *Fr* = 7.982. This condition is strongly inertia-dominated. In spray drying, this indicates that the atomizing gas momentum is much stronger than gravity. This condition can improve droplet dispersion, but it may also increase the risk of wall impingement, particle loss, non-uniform residence time, and unstable drying behavior.

**Table 15.** Input data for industrial *Fr* number examples.

| **CASE** | **INDUSTRIAL APPLICATION** | **v (m/s)** | **g (m/s²)** | **L (m)** |
|---|---|---|---|---|
| 1 | Wastewater flow in an open channel | 0.80 | 9.81 | 0.5 |
| 2 | Low-velocity spray region in a spray dryer | 2.00 | 9.81 | 1.2 |
| 3 | Critical spray region in a spray dryer | 3.43 | 9.81 | 1.2 |
| 4 | Critical flow condition in a channel or vessel | 3.13 | 9.81 | 1.0 |
| 5 | High-speed liquid flow in a mixing vessel | 4.50 | 9.81 | 0.8 |
| 6 | High-velocity drying-air jet near an atomizer | 12.00 | 9.81 | 1.0 |
| 7 | Very high-velocity atomizing gas in a spray dryer | 25.00 | 9.81 | 1.0 |

**Table 16.** Summary of *Fr* results and engineering handling strategies.

| CASE | INDUSTRIAL APPLICATION | *Fr* Value | FLOW CONDITION | INDUSTRIAL INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|
| 1 | Wastewater flow in an open channel | 0.361 | Gravity-dominated flow | The flow is mainly controlled by gravity. Surface disturbance is relatively low. | Maintain channel slope and flow depth. Avoid sediment accumulation and sudden changes in channel geometry. |
| 2 | Low-velocity spray region in a spray dryer | 0.583 | Gravity-dominated flow | Gravity still strongly affects the spray or droplet trajectory. Droplets may move downward more easily. | Check droplet residence time, drying-air distribution, and wall deposition risk. Increase gas velocity or improve the atomizer position if droplets settle too quickly. |
| 3 | Critical spray region in a spray dryer | 1.000 | Critical condition | Gas momentum and gravity are nearly balanced. The droplet trajectory may change easily when operating conditions change. | Control drying-air velocity, atomizer position, droplet size, and chamber geometry carefully. Avoid sudden changes in gas flow rate. |
| 4 | Critical flow condition in a channel or vessel | 1.000 | Critical condition | Inertial and gravitational effects are balanced. Small changes in velocity or liquid level may change the flow behavior. | Control operating velocity and liquid level carefully. Avoid sudden changes in flow rate or geometry. |
| 5 | High-speed liquid flow in a mixing vessel | 1.606 | Inertia-dominated flow | Inertial force dominates. Strong vortex, splashing, or surface depression may occur. | Reduce the impeller speed if an excessive vortex appears. Install baffles, improve tank geometry, or adjust liquid level. |
| 6 | High-velocity drying-air jet near an atomizer | 3.831 | Inertia-dominated flow | The gas jet strongly dominates over gravity. The droplet trajectory is mainly controlled by gas momentum. | Check atomizer design, gas velocity, chamber size, wall deposition, and particle residence time. Reduce jet velocity or adjust flow distribution if wall impingement occurs. |
| 7 | Very high-velocity atomizing gas in a spray dryer | 7.982 | Strongly inertia-dominated flow | Atomizing gas momentum is much stronger than gravity. Droplet dispersion may increase, but wall impingement and particle loss may also increase. | Optimize atomizing gas pressure, reduce excessive jet velocity, adjust spray angle, increase chamber diameter if needed, and check wall deposition and cyclone loading. |

When *Fr* is lower than 1, gravity has a stronger influence on the flow. This condition is commonly found in open-channel flow, wastewater treatment ponds, and low-velocity spray regions:

(i) In wastewater treatment ponds or channels, low *Fr* usually indicates relatively stable surface flow, but sediment accumulation may occur if the flow velocity is too low. Therefore, channel slope, flow depth, hydraulic residence time, and sludge accumulation should be monitored.
(ii) In spray drying, this condition may increase the possibility of droplet settling, short residence time, or wet wall deposition if the drying-air momentum is insufficient.
(iii) In mixing vessels, a low *Fr* indicates that gravitational effects are stronger than inertial effects. This condition usually produces a more stable liquid surface with less vortex formation, splashing, and air entrainment. Therefore, low *Fr* may be suitable for gentle mixing processes where surface stability is required. However, if the impeller speed is too low, mixing intensity may be insufficient, and dead zones may form inside the vessel.
(iv) In slurry systems, low *Fr* may also increase the risk of particle settling because the mixing energy is not strong enough to keep particles suspended. Therefore, low-*Fr* mixing conditions should be evaluated based on the process objective. If stronger mixing is required, the impeller speed can be increased gradually, a more suitable impeller can be selected, or baffles can be used to improve flow circulation without producing excessive vortex.

When *Fr* is approximately equal to 1, inertial and gravitational effects are nearly balanced. This condition is sensitive because small changes in velocity, liquid level, flow depth, atomizer position, or system geometry may change the flow behavior. In open-channel or wastewater systems, this may affect surface stability and flow distribution. In spray drying, it may influence droplet trajectory and residence behavior. Therefore, operating velocity and system geometry should be controlled carefully near the critical condition. When *Fr* is higher than 1, inertial effects become more dominant. In wastewater treatment ponds or open-channel systems, a high *Fr* indicates that inertial effects are stronger than gravitational effects. This condition may produce faster and more aggressive flow, stronger surface disturbance, and less stable hydraulic behavior. Although higher inertia can improve mixing and flow distribution, excessive *Fr* may reduce hydraulic residence time and cause hydraulic short-circuiting, where wastewater moves too quickly from the inlet to the outlet. This can decrease treatment efficiency because sedimentation, biological degradation, and contact time become insufficient. High *Fr* may also resuspend settled sludge, increase erosion near the inlet or channel bed, and disturb the separation zone. Therefore, high-*Fr* conditions in wastewater ponds should be controlled by reducing inlet velocity, improving inlet and outlet design, using baffles or flow-distribution structures, increasing pond length or effective flow path, and preventing direct short-circuit flow between the inlet and outlet. In mixing vessels, high *Fr* may produce stronger surface motion, vortex formation, air entrainment, splashing, and surface depression. These effects may disturb mixing performance and reduce process stability. One common engineering strategy is to install baffles. Baffles reduce tangential swirling motion, suppress vortex formation, improve axial and radial mixing, and help distribute energy more effectively inside the vessel. If excessive vortex or air entrainment occurs, the impeller speed can also be reduced, the liquid level can be adjusted, or the impeller position and tank geometry can be improved. In spray drying systems, a high *Fr* near the atomizer indicates that the gas jet or spray momentum is stronger than gravity. This condition can help disperse droplets and carry them into the drying chamber. However, excessive gas momentum may also increase wall impingement, non-uniform residence time, particle loss, noise, and unstable flow distribution. Therefore, atomizer position, spray angle, drying-air velocity, chamber diameter, and air distributor design should be optimized.

*Fr* can be used as a practical tool for evaluating the relative importance of gravity and inertia in different industrial flow systems. In wastewater treatment ponds and channels, it helps evaluate flow stability, sedimentation tendency, and hydraulic behavior. In mixing vessels, it helps assess vortex formation, free-surface motion, and the need for baffles. In spray drying systems, it helps determine whether droplet movement is mainly controlled by gravity or gas momentum. However, *Fr* number should be combined with other dimensionless numbers, such as *Re*, *We*, and *St* numbers, to obtain a more complete understanding of flow regime, droplet breakup, particle response, and overall process behavior.

### 3.1.5. Euler Number: Pressure Drop and Energy Loss in Industrial Flow Systems

Euler number, *Eu*, is used to compare the pressure force with the inertial force in a fluid-flow system. In chemical engineering applications, *Eu* number is useful for evaluating pressure drop, flow resistance, and energy loss in pipes, valves, fittings, filters, nozzles, heat exchangers, cyclones, air distributors, and other process equipment.

Unlike *Re*, *Ma*, and *Fr* numbers, the *Eu* number does not have a universal threshold for classifying flow behavior. *Eu* is mainly used as a comparative indicator of pressure loss relative to flow inertia. Therefore, the interpretation of *Eu* depends on equipment type, design baseline, allowable pressure drop, and operating conditions. The practical interpretation of the *Eu* number is presented in **Table 17**.

**Table 17.** Practical interpretation of *Eu* number.

| EU CONDITION | PRACTICAL INTERPRETATION | POSSIBLE ENGINEERING MEANING |
|---|---|---|
| Low Eu | Pressure loss is relatively small | Flow resistance is low |
| Moderate Eu | Pressure loss should be considered in design | Pump or blower load may increase |
| High Eu | Pressure loss is significant | Energy demand and operating cost may increase |
| Increasing Eu over time | Flow resistance is increasing | Fouling, blockage, valve restriction, or filter clogging may occur |

*Eu* is most useful in systems where pressure drop or flow resistance is important. The common applications of the *Eu* number in chemical engineering systems are summarized in **Table 18**. *Eu* number is more suitable for pressure-driven systems than for open free-surface systems. In mixing vessels, *Eu* is not usually the primary dimensionless number for impeller performance because *Re*, *Fr*, and *Np* numbers are more commonly used. However, *Eu* number can still be useful when a pressure drop occurs in recirculation lines, inlet nozzles, spargers, gas distributors, or external piping connected to the vessel. In wastewater ponds, *Eu* is also not the main parameter for describing pond hydraulics because *Fr* number and *Re* number are usually more relevant. However, *Eu* can still be applied to pump lines, inlet pipes, outlet structures, valves, filters, or aeration diffuser systems.

**Table 18.** Common applications of *Eu* number in industrial systems.

| SYSTEM | USEFULNESS OF *Eu* | ENGINEERING REASON |
|---|---|---|
| Pipe flow | High | To evaluate the pressure drop relative to the flow inertia |
| Valve and fitting | High | To evaluate local pressure loss |
| Filter or membrane unit | High | To detect clogging, fouling, or increasing resistance |
| Cyclone and ducting system | High | To evaluate gas pressure loss and blower load |
| Spray drying system | High | To evaluate the drying-air duct, air distributor, cyclone, bag filter, and atomizing gas line |
| Heat exchanger | High | To evaluate the pressure drop across tubes, shells, or channels |
| Mixing vessel | Limited | Useful mainly for the inlet, outlet, sparger, nozzle, or external circulation line |
| Wastewater pond | Limited | Less common for open pond flow, but useful for inlet pipe, outlet structure, pump line, valve, or diffuser |

To show the practical use of *Eu* number, five industrial examples are presented in **Table 19**. The calculation results, industrial interpretation, and engineering handling strategies are summarized in **Table 20**. The examples are arranged from low *Eu* to high *Eu* to show how pressure loss becomes more important as *Eu* increases:

(i) For the first case, cooling water flows through a straight pipe, resulting in *Eu* = 0.125. Pressure loss is relatively small compared with flow inertia.

(ii) For the second case, drying air flows through an air distributor in a spray dryer, resulting in *Eu* = 1.422. Pressure loss should be considered in the air distribution system.

(iii) For the third case, gas flows through a process pipeline with several fittings, resulting in *Eu* = 3.125. Pressure loss is significant and may affect blower or compressor capacity.

(iv) For the fourth case, liquid flows through a control valve, resulting in *Eu* = 5.000. The valve produces a large local pressure loss.

(v) For the fifth case, slurry flows through a partially blocked pipe or filter, resulting in *Eu* = 8.081. This indicates very high-pressure loss, which may be caused by blockage, fouling, filter clogging, or high slurry resistance.

**Table 19.** Input data for industrial *Eu* number examples.

| CASE | INDUSTRIAL APPLICATION | $\Delta P$ (Pa) | $\rho$ (kg/m$^3$) | $v$ (m/s) |
|---|---|---|---|---|
| 1 | Cooling water flow in a straight pipe | 500 | 1000 | 2.00 |
| 2 | Drying-air flow through an air distributor in a spray dryer | 800 | 0.90 | 25.00 |
| 3 | Gas flow through a process pipeline with fittings | 1500 | 1.20 | 20.00 |
| 4 | Liquid flow through a control valve | 20000 | 1000 | 2.00 |
| 5 | Slurry flow through a partially blocked pipe or filter | 80000 | 1100 | 3.00 |

**Table 20.** Summary of *Eu* number results and engineering handling strategies.

| CASE | INDUSTRIAL APPLICATION | *Eu* VALUE | PRESSURE LOSS CONDITION | INDUSTRIAL INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|
| 1 | Cooling water flow in a straight pipe | 0.125 | Low | Pressure loss is small, and the flow system is relatively efficient. | Maintain normal operation. Monitor pressure drop periodically and keep the pipe clean. |
| 2 | Drying-air flow through an air distributor in a spray dryer | 1.422 | Moderate | Air distributor resistance affects drying-air flow and chamber distribution. | Check air distributor design, blower capacity, pressure drop, and drying-air uniformity. |
| 3 | Gas flow through a process pipeline with fittings | 3.125 | Significant | Pipe length, bends, and fittings may increase blower or compressor load. | Reduce unnecessary fittings, use smoother bends, increase pipe diameter if needed, and check blower capacity. |
| 4 | Liquid flow through a control valve | 5.000 | High | The valve produces a large local pressure loss and increases pumping energy. | Select a suitable valve size, check valve opening, avoid excessive throttling, and evaluate pump capacity. |
| 5 | Slurry flow through a partially blocked pipe or filter | 8.081 | Very high | Fouling, blockage, or solid accumulation strongly increases pressure loss. | Inspect and clean the pipe or filter, reduce blockage, maintain suitable slurry velocity, and use appropriate pump capacity. |

The *Eu* number helps evaluate how significant the pressure loss is relative to flow inertia. A low *Eu* indicates that pressure loss is relatively small, while a high *Eu* indicates that the system requires more energy to maintain the flow. In industrial systems, high *Eu* may occur because of small pipe diameter, long pipe length, excessive fittings, partially closed valves, fouling, blockage, filter clogging, or high slurry resistance.

In spray drying systems, *Eu* number is useful for evaluating pressure loss in drying-air lines, air distributors, atomizing gas lines, cyclones, bag filters, and exhaust ducts. A moderate or high *Eu* may indicate that the blower must overcome significant flow resistance. If the pressure drop is too high, drying-air distribution may become non-uniform, energy consumption may increase, and drying performance may become unstable.

In heat exchangers, *Eu* number can be used to compare pressure drop across tubes, shells, or flow channels. A high *Eu* may indicate excessive hydraulic resistance, fouling, or poor flow-path design. In wastewater treatment systems, *Eu* number is more relevant for pump lines, inlet pipes, outlet structures, valves, and aeration diffusers than for the open pond itself. In mixing vessels, *Eu* number is useful mainly when pressure-driven flow exists, such as in spargers, recirculation loops, inlet nozzles, or external piping.

In addition to design comparison, *Eu* number can also be used for operational monitoring. Since *Eu* number is related to pressure drop, an increase in *Eu* under similar flow conditions may indicate that flow resistance is increasing. This situation may occur due to fouling, blockage, filter clogging, particle deposition, valve restriction, or solid accumulation in the equipment. For monitoring purposes, the current *Eu* number can be compared with the initial or clean-condition *Eu* number.

(i) The clean-condition *Eu* number is written as: $Eu_{initial} = \Delta P_{initial}/(\rho v^2)$.

(ii) The current *Eu* number during operation is written as: $Eu_{current} = \Delta P_{current}/(\rho v^2)$.

(iii) The increase ratio is calculated as $Eu_{ratio} = Eu_{current}/Eu_{initial}$.

If fluid density and velocity remain approximately constant, the *Eu* number ratio becomes directly proportional to the pressure-drop ratio: $Eu_{ratio} = \Delta P_{current}/\Delta P_{initial}$. This comparison is useful because *Eu* number does not have a universal threshold. Therefore, the acceptable value should be determined based on the initial clean condition, equipment design limit, allowable pressure drop, and operating experience.

A practical example of *Eu* number monitoring in a spray drying air-filter system is presented in **Table 21**. In this example, drying air passes through a filter or duct system at nearly constant density and velocity. As the system operates, dust accumulation increases the pressure drop. The *Eu* number increases from 1.422 to 4.622. The value after three weeks is about 3.25 times higher than the clean-condition value. The system resistance has increased significantly. In a spray drying system, this condition may be caused by dust accumulation in the filter, blockage in the air duct, particle deposition, or increasing resistance in the exhaust line. The engineering handling strategy is to inspect the filter, duct, cyclone, or bag filter system. If dust accumulation is confirmed, cleaning or filter replacement should be performed. The blower capacity should also be checked because increasing pressure drop increases energy demand and may reduce drying-air flow rate. If the problem occurs frequently, the filter area, duct design, cyclone efficiency, or cleaning schedule should be improved.

**Table 21.** Monitoring of *the Eu* number over time in a spray drying air-filter system.

| TIME | ΔP (Pa) | $\rho$ (kg/m³) | *v* (m/s) | *Eu* VALUE | *Eu* RATIO | POSSIBLE CONDITION |
|---|---|---|---|---|---|---|
| Initial clean condition | 800 | 0.90 | 25.00 | 1.422 | 1.00 | Clean system |
| After 1 week | 1200 | 0.90 | 25.00 | 2.133 | 1.50 | Resistance starts to increase. |
| After 2 weeks | 1800 | 0.90 | 25.00 | 3.200 | 2.25 | Possible dust accumulation or filter loading |
| After 3 weeks | 2600 | 0.90 | 25.00 | 4.622 | 3.25 | High resistance; inspection or cleaning is required |

A practical interpretation of *Eu* number monitoring is summarized in **Table 22**. The values should not be treated as universal limits. They are practical monitoring guidelines. The actual action limit should be adjusted based on equipment design, allowable pressure drop, safety margin, product characteristics, and operating experience. Therefore, *Eu* number can be used as a practical tool for pressure-drop evaluation, design comparison, and troubleshooting. It helps engineers compare different equipment designs, monitor increasing flow resistance, identify possible fouling or blockage, and evaluate whether pumps or blowers are sufficient for stable operation. By comparing $Eu_{current}$ with $Eu_{initial}$, engineers can detect increasing flow resistance and take preventive action before the system experiences severe pressure drop, unstable operation, or excessive energy consumption.

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**Table 22.** Practical use of the *Eu* ratio for operational monitoring.

| ***Eu* RATIO** | **PRACTICAL INTERPRETATION** | **ENGINEERING ACTION** |
|---|---|---|
| Around 1.0 | Similar to clean-condition operation | Continue normal monitoring |
| 1.2-1.5 | Flow resistance starts to increase | Check the pressure trend and operating condition |
| 1.5-2.0 | The resistance increase becomes significant | Plan inspection or cleaning |
| More than 2.0 | High possibility of fouling, blockage, or filter clogging | Inspect and clean the system; check the pump or blower load |

### 3.1.6. Power Number: Mixing Power Requirement in Industrial Agitation Systems

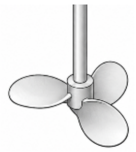
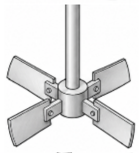
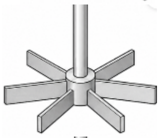
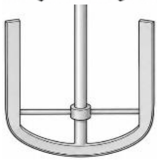
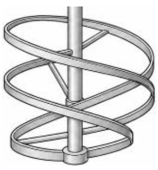
Power number, *Np*, is used to evaluate the power requirement in mixing and agitation systems. In chemical engineering applications, *Np* is important for stirred tanks, impeller selection, mixing scale-up, slurry suspension, gas-liquid dispersion, blending tanks, fermentation systems, wastewater mixing basins, and feed preparation tanks before spray drying. It relates the actual power input to the inertial mixing scale of the fluid. Unlike *Re* or *Ma*, this *Np* number does not have a universal threshold for classifying mixing conditions. The value of *Np* depends strongly on impeller type, tank geometry, baffle configuration, fluid properties, flow regime, solid loading, gas dispersion, and scale-up condition. Therefore, *Np* is mainly used as a design and comparison parameter rather than as a fixed classification parameter. In mixing systems, *Np* is commonly interpreted by comparing the calculated value with typical values for a specific impeller and tank configuration. In fully turbulent mixing with similar geometry, *Np* often becomes nearly constant for a given impeller type. Therefore, *Np* is useful for estimating mixing power, comparing impeller performance, selecting motor capacity, and supporting scale-up. The practical interpretation of *Np* is presented in **Table 23**.

**Table 23.** Practical interpretation of *Np* number.

| **NP CONDITION** | **PRACTICAL INTERPRETATION** | **POSSIBLE ENGINEERING MEANING** |
|---|---|---|
| Low *Np* | Low power demand | Mixing energy requirement is small, but mixing may be weak |
| Moderate *Np* | Moderate power demand | Mixing may be acceptable if homogeneity is achieved |
| High *Np* | High power demand | Energy consumption and operating cost may increase |
| Increasing *Np* | Mixing resistance increases | Fluid viscosity, impeller type, tank geometry, or solid loading may affect power demand |
| Constant *Np* at high *Re* | Fully turbulent mixing condition | Power can be estimated for scale-up using a similar geometry |

Typical *Np* values for several impeller types are shown in **Table 24**. These values should not be treated as strict thresholds, but as reference ranges for interpreting *Np* based on impeller type. A high *Np* value does not always indicate poor performance, because some impellers, such as Rushton turbines, naturally require higher power to produce strong radial flow and gas dispersion. Similarly, a low *Np* value does not always indicate good performance, because low power input may result in slow mixing, poor suspension, or non-uniform concentration. Therefore, *Np* should be evaluated together with the mixing objective, power consumption, fluid viscosity, baffle configuration, and required process performance.

**Table 24.** Typical *Np* range based on impeller type.

| IMPELLER TYPE | TYPICAL *Np* RANGE | GENERAL INTERPRETATION |
|---|---|---|
| Marine propeller  | 0.3-0.6 | Low power demand; suitable for axial circulation |
| Pitched-blade turbine  | 1-2 | Moderate power demand; suitable for general mixing |
| Rushton turbine  | 4-6 | High power demand, suitable for strong radial mixing and gas dispersion |
| Anchor impeller  | Variable | Used for viscous fluids; it depends strongly on viscosity and clearance |
| Helical ribbon impeller  | Variable | Used for high-viscosity mixing; geometry strongly affects power demand |

*Np* is mainly used in mechanically agitated systems. Its usefulness in different industrial systems is summarized in **Table 25**. *Np* is most relevant when mechanical agitation is used. In mixing vessels, *Np* is one of the main parameters for estimating power consumption. In slurry systems, it helps evaluate whether the impeller provides enough energy to keep particles suspended. In spray drying, *Np* is not usually used to analyze the drying chamber itself. However, it is useful for the feed preparation tank, where the liquid feed, slurry, or emulsion must be mixed before atomization. In wastewater systems, *Np* can be applied when mechanical mixers or surface aerators are used, but it is less relevant for open ponds without mechanical agitation.

Baffles also influence mixing performance and *Np*. In an unbaffled tank, the liquid may rotate with the impeller and form a vortex. This condition can reduce effective mixing because much of the energy is used to produce swirling motion instead of axial or radial circulation. Installing baffles can reduce tangential swirling, suppress vortex formation, improve flow circulation, and increase mixing effectiveness. However, baffles may also increase power demand because they increase resistance to fluid motion. Therefore, baffle configuration should be considered when interpreting *Np* and designing mixing vessels.

**Table 25.** Common applications of the *Np* number in industrial systems.

| SYSTEM | USEFULNESS OF *Np* | ENGINEERING REASON |
|---|---|---|
| Mixing vessel | High | To estimate agitation power and select the impeller type |
| Slurry tank | High | To evaluate the power required to suspend solid particles |
| Gas-liquid stirred tank | High | To support gas dispersion and mixing design |
| Fermentation tank | High | To evaluate agitation energy and oxygen transfer support |
| Spray drying feed tank | Moderate | Useful for feed preparation and slurry or emulsion homogenization before atomization |
| Wastewater aeration or mixing basin | Moderate | Useful when mechanical mixers or surface aerators are used |
| Heat exchanger | Low | Not usually used unless the system includes a stirred vessel or a mechanically agitated unit |
| Open wastewater pond without mechanical mixing | Low | The *Fr* number and the *Re* number are usually more relevant |

To show the practical use of *Np*, five industrial examples are presented in **Table 26**. The examples are arranged from low *Np* to high *Np* using $Np = \frac{P_w}{\rho_{mix} N^3 D_{imp}^5}$ to show how the mixing power demand increases under different operating conditions.

The calculation results, industrial interpretation, and engineering handling strategies are summarized in **Table 27**.

*Np* helps compare the power demand of different mixing systems. A low *Np* indicates low power consumption, but it does not always mean that the mixing process is effective. If the mixing intensity is too weak, concentration gradients, temperature differences, or particle settling may occur. A high *Np* indicates that more power is required, which may increase operating cost and motor load.

**Table 26.** Input data for industrial *Np* number examples.

| CASE | INDUSTRIAL APPLICATION | *Pw* (W) | $\rho$ (kg/m³) | *N* (1/s) | $D_{imp}$ (m) | *Np* |
|---|---|---|---|---|---|---|
| 1 | Low-speed blending of liquid in a tank | 80 | 1000 | 5.00 | 0.35 | 0.122 |
| 2 | Feed mixing tank before spray drying | 120 | 1050 | 5.00 | 0.30 | 0.376 |
| 3 | Wastewater mixing basin with mechanical agitator | 350 | 1000 | 4.00 | 0.40 | 0.534 |
| 4 | Slurry suspension in a stirred tank | 900 | 1200 | 4.50 | 0.45 | 0.446 |
| 5 | High-viscosity liquid mixing in a process tank | 1500 | 1300 | 3.50 | 0.40 | 2.627 |

**Table 27.** Summary of *Np* number results and engineering handling strategies.

| CASE | INDUSTRIAL APPLICATION | *Np* VALUE | POWER CONDITION | INDUSTRIAL INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|
| 1 | Low-speed blending of liquid in a tank | 0.122 | Low | Mixing power demand is small, but mixing may be slow. | Continue operation if homogeneity is achieved. Increase impeller speed if mixing time is too long. |
| 2 | Feed mixing tank before spray drying | 0.376 | Moderate | Feed preparation power is acceptable if the slurry or emulsion remains uniform before atomization. | Check feed homogeneity, viscosity, solid suspension, and atomizer feed stability. Adjust the impeller speed if separation occurs. |
| 3 | Wastewater mixing basin with mechanical agitator | 0.534 | Moderate | Agitation helps distribute solids, oxygen, or chemicals in the basin. | Check dead zones, sludge settling, and mixing uniformity. Adjust mixer location or speed if circulation is poor. |
| 4 | Slurry suspension in a stirred tank | 0.446 | Moderate | Mixing power may be sufficient, but particle settling must be checked. | Maintain suitable impeller speed, select proper impeller type, avoid dead zones, and check suspension quality. |
| 5 | High-viscosity liquid mixing in a process tank | 2.627 | High | High mixing resistance increases power demand and operating cost. | Use a suitable impeller for viscous fluids, reduce viscosity if possible, optimize speed, and evaluate motor capacity. |

In spray drying systems, *Np* is useful mainly in the feed preparation stage. Before atomization, the feed solution, slurry, or emulsion should be sufficiently mixed to maintain uniform concentration, viscosity, and solid distribution. If the feed is not homogeneous, atomization may become unstable, and particle quality may vary. Therefore, the mixing tank before the spray dryer should be evaluated using *Np*, together with viscosity, solid content, and feed stability. In wastewater treatment systems, *Np* is useful when mechanical mixers or surface aerators are used. Low mixing power may cause dead zones, sludge settling, and poor chemical distribution. Excessive mixing power may increase energy consumption and disturb sedimentation zones. Therefore, the mixer speed, impeller type, and mixer position should be adjusted based on the treatment objective. In slurry and high-viscosity systems, *Np* should be evaluated together with the *Re* number and power consumption. A suitable impeller should be selected to produce enough circulation without excessive energy use. Baffles may also be used in mixing vessels to reduce vortex formation and improve axial and radial mixing. However, in highly viscous systems, baffle design and impeller type should be selected carefully because excessive resistance can increase power demand. Therefore, *Np* can be used as a practical tool for estimating mixing power, comparing impeller performance, selecting motor capacity, and supporting scale-up. It is especially important in stirred tanks, slurry systems, feed preparation tanks, fermentation processes, and wastewater mixing systems.

In addition, baffles are commonly installed in mixing vessels to reduce tangential swirling motion and suppress vortex formation. Without baffles, the liquid may rotate together with the impeller, causing poor axial and radial circulation. With baffles, the flow becomes more turbulent and better distributed, but the required power input may increase. Therefore, the effect of baffles can be evaluated by comparing the Power number before and after baffle installation. The input data for a mixing vessel with and without baffles are presented in **Table 28**. The comparison of *Np* with and without baffles is summarized in **Table 29**. Installing baffles increases the *Np* from 0.381 to 0.687 in this example. The mixing system requires more power after baffle installation. However, the higher power demand is not always negative because baffles improve mixing effectiveness by reducing vortex formation and increasing flow circulation inside the vessel. Therefore, the effect of baffles should be evaluated based on the process objective. If the goal is gentle mixing with low energy consumption, an unbaffled tank may be acceptable. However, if the process requires uniform mixing, better solid suspension, reduced vortex, or improved heat and mass transfer, baffles are usually beneficial. The motor capacity should also be checked because baffled tanks may require higher power input than unbaffled tanks.

**Table 28.** Input data for *Np* comparison with and without baffles.

| CASE | MIXING CONDITION | *Pw* (W) | $\rho$ (kg/m³) | *N* (1/s) | $D_{imp}$ (m) |
|---|---|---|---|---|---|
| 1 | Mixing vessel without baffles | 250 | 1000 | 4.00 | 0.40 |
| 2 | Mixing vessel with baffles | 450 | 1000 | 4.00 | 0.40 |

**Table 29.** Effect of baffles on *Np* number and mixing performance.

| CASE | MIXING CONDITION | *Np* VALUE | INDUSTRIAL INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|
| 1 | Without baffles | 0.381 | Lower power demand, but swirling and vortex formation may occur. Mixing may be less effective because the liquid rotates with the impeller. | Use this condition only when gentle mixing is required. If vortex, poor mixing, or dead zones occur, install baffles or adjust impeller position. |
| 2 | With baffles | 0.687 | Power demand increases, but vortex formation is reduced, and axial/radial circulation improves. Mixing becomes more effective. | Use baffles when uniform mixing, solid suspension, or better circulation is required. Check motor capacity because power demand may increase. |

### 3.1.7. Stokes Number: Particle Response in Industrial Fluid Flow Systems

Stokes number, *Stk*, is used to evaluate whether particles or droplets can follow the motion of the surrounding fluid. In chemical engineering applications, *Stk* is important for slurry transport, wastewater treatment, spray drying, aerosol flow, cyclone separation, sedimentation, and pneumatic conveying systems. *Stk* number is especially useful when particles, droplets, or solids are transported by gas or liquid flow, as summarized in **Table 30**. In spray drying, *Stk* helps evaluate whether droplets follow the drying-air stream or deviate toward the chamber wall. In wastewater treatment, it helps evaluate whether flocs or suspended particles move with the water or tend to settle. In slurry and mixing systems, it helps determine whether mixing intensity is sufficient to keep particles suspended.

**Table 30.** Common applications of the *Stk* number in industrial systems.

| SYSTEM | USEFULNESS OF *Stk* | ENGINEERING REASON FOR EVALUATION |
|---|---|---|
| Spray drying | High | The droplet or particle following behavior in drying air |
| Aerosol transport | High | Whether fine particles follow the gas flow |
| Cyclone separator | High | particle separation tendency |
| Slurry flow | High | Particle suspension and settling tendency |
| Wastewater treatment | Moderate to high | Floc, sludge, or particle transport and settling |
| Mixing vessel with solids | High | Whether particles remain suspended or settle |
| Pneumatic conveying | High | Particle response to gas flow direction changes |
| Heat exchanger with fouling particles | Moderate | Possible particle deposition on surfaces |

The *Stk* is defined as: $Stk = \frac{\tau_p}{\tau_f}$. For small spherical particles under the Stokes drag condition, the particle response time can be estimated using correlation $\tau_p = \frac{\rho_p d_p^2}{18\mu_f}$ , and the fluid characteristic time can be estimated using correlation $\tau_f = \frac{L_c}{v_c}$. The general interpretation of the *Stk* is presented in **Table 31**. Unlike the *Re* (which is used to classify flow regime), *Stk* is mainly used to evaluate particle-fluid interaction. A low *Stk* indicates that particles or droplets can follow the fluid streamlines easily. A high *Stk* indicates that particles have stronger inertia and may deviate from the flow path, settle, impact equipment walls, or separate from the gas or liquid stream.

**Table 31.** General interpretation of the *Stk* number.

| STOKES NUMBER CONDITION | PARTICLE BEHAVIOR | GENERAL MEANING |
|---|---|---|
| *Stk* << 1 | Particle follows fluid motion | Particle inertia is weak |
| *Stk* ≈ 1 | Particle partially follows fluid motion | Particle inertia becomes important |
| *Stk* >> 1 | Particle does not easily follow fluid motion | Particle motion is dominated by its own inertia |

To show the practical use of *Stk*, industrial examples are presented in **Table 32**. The examples are arranged from low to high *Stk* value to show how particle behavior changes as particle inertia increases.

The calculation results, industrial interpretation, and engineering handling strategies are summarized in **Table 33**. *Stk* helps compare particle-following behavior in different industrial systems. When *Stk* is much lower than 1, particles or droplets follow the fluid motion closely. This condition is common for fine aerosol droplets in spray drying and fine flocs in wastewater flow. When *Stk* approaches 1, particle inertia becomes important, and particles may partially deviate from the flow path. When *Stk* is much higher than 1, particles are dominated by their own inertia and may not follow the fluid streamlines.

**Table 32.** Input data for industrial *Stk* number examples.

| CASE | INDUSTRIAL APPLICATION | $\rho_p$ (kg/m³) | *dp* (m) | $\mu f$ (Pa.s) | *Lc* (m) | *vc* (m/s) | *Stk* |
|---|---|---|---|---|---|---|---|
| 1 | Fine floc transport in wastewater flow | 1,050 | $2.0 \times 10^{-5}$ | $1.0 \times 10^{-3}$ | 0.20 | 0.10 | 0.00001 |
| 2 | Fine aerosol droplet in spray drying | 1,000 | $5.0 \times 10^{-6}$ | $2.0 \times 10^{-5}$ | 0.50 | 20.00 | 0.00278 |
| 3 | Fine particle in slurry flow | 2,650 | $1.0 \times 10^{-4}$ | $1.0 \times 10^{-3}$ | 0.01 | 2.00 | 0.29400 |
| 4 | Critical particle response in liquid flow | 2,500 | $1.2 \times 10^{-4}$ | $1.0 \times 10^{-3}$ | 0.01 | 5.00 | 1.00000 |
| 5 | Coarse droplet in spray drying | 1,000 | $1.5 \times 10^{-4}$ | $2.0 \times 10^{-5}$ | 0.50 | 20.00 | 2.50000 |
| 6 | Coarse dust particle in cyclone gas flow | 1,500 | $2.0 \times 10^{-4}$ | $1.8 \times 10^{-5}$ | 0.10 | 15.00 | 27.8000 |

**Table 33.** Summary of *Stk* number results and engineering handling strategies.

| CASE | INDUSTRIAL APPLICATION | *Stk* VALUE | PARTICLE RESPONSE CONDITION | INDUSTRIAL INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|
| 1 | Fine floc transport in wastewater flow | $1.17 \times 10^{-5}$ | Very low | Fine flocs follow the wastewater flow easily. | Maintain suitable hydraulic flow and avoid excessive turbulence that may break flocs. |
| 2 | Fine aerosol droplet in spray drying | 0.00278 | Low | Fine droplets follow the drying-air streamlines. | Maintain stable drying-air distribution and avoid excessive wall-directed flow. |
| 3 | Fine particle in slurry flow | 0.294 | Low to moderate | Particles mostly follow the liquid flow, but inertia is still present. | Maintain sufficient mixing or flow velocity to avoid settling and non-uniform slurry distribution. |
| 4 | Critical particle response in liquid flow | 1.000 | Critical response | Particle inertia and fluid motion are comparable; particles may partly deviate from the flow. | Adjust particle size, flow velocity, or mixing intensity to control particle trajectory and suspension. |
| 5 | Coarse droplet in spray drying | 2.500 | High | Coarse droplets may deviate from the drying-air flow and impact the dryer wall. | Reduce droplet size, optimize atomizer pressure, adjust spray angle, improve chamber airflow, and check wall deposition. |
| 6 | Coarse dust particle in cyclone gas flow | 27.8 | Very high | Particles do not follow gas streamlines easily and may be separated by inertia. | Use cyclone or separator design properly, control inlet velocity, and check collection efficiency and erosion risk. |

In spray drying systems, low-*Stk* droplets generally follow the drying-air flow and are easier to carry through the chamber. However, high-*Stk* droplets are more difficult to redirect with the gas flow and may impact the chamber wall, leading to wet deposition, wall sticking, non-uniform drying, or product loss. Therefore, droplet size, atomizer condition, spray angle, chamber geometry, and drying-air distribution should be controlled carefully.

In wastewater treatment and slurry systems, *Stk* helps evaluate whether particles or flocs follow the liquid flow or tend to settle. Low-*Stk* particles are easier to transport, while higher-*Stk* particles may require stronger mixing, higher flow velocity, or improved hydraulic design to prevent sedimentation and dead zones. In cyclone separation, high *Stk* can be useful because particles with strong inertia are more easily separated from the gas stream.

*Stk* number can be used as a practical tool for evaluating particle response, droplet transport, wall deposition risk, particle settling, and separation behavior. However, the simple particle response equation assumes small spherical particles and Stokes drag conditions. For irregular particles, concentrated suspensions, high particle Re, or non-Newtonian fluids, correction factors or more detailed models may be required.

### 3.1.8. Weber Number: Droplet, Bubble, and Interfacial Deformation in Industrial Flow Systems

Weber number, *We*, is used to compare the inertial force with the surface tension force in a fluid system. This dimensionless number is important in chemical engineering because many industrial processes involve droplets, bubbles, liquid jets, sprays, foams, emulsions, and gas-liquid or liquid-liquid interfaces. For a more general system, the characteristic length, *Lc*, can also be used. But, in droplet and bubble systems, $d_b$ is commonly used as the characteristic length. In jet or free-surface systems, *Lc* may represent the jet diameter, nozzle diameter, or another relevant characteristic length.

The general interpretation of the *We* is presented in **Table 34**. A low *We* indicates that surface tension is strong enough to maintain the shape of droplets or bubbles. A high *We* indicates that the inertial force is strong enough to deform or break the interface. However, the *We* should not be interpreted using one universal threshold only. The actual deformation or breakup condition also depends on viscosity, density ratio, turbulence, nozzle design, gas velocity, liquid properties, and residence time. The *We* is useful in many industrial systems, as summarized in **Table 35**. The *We* is especially useful when the process involves an interface between two phases. In spray drying, the *We* number can be used to evaluate whether droplets are stable or likely to deform and break. In wastewater aeration, it can help evaluate bubble deformation and gas-liquid contact behavior. In emulsion and mixing systems, the We can indicate whether the mixing energy is sufficient to break droplets into smaller sizes.

**Table 34.** General interpretation of the *We* number.

| CONDITION | DOMINANT FORCE | GENERAL MEANING |
|---|---|---|
| We << 1 | Surface tension dominates | Droplets or bubbles tend to remain stable and spherical |
| We ≈ 1 | Inertia and surface tension are comparable | Droplets or bubbles may start to deform |
| We > 1 | Inertia becomes important | Droplet or bubble deformation becomes significant |
| We >> 1 | Inertia strongly dominates | Breakup, atomization, or strong interfacial deformation may occur |

**Table 35.** Common applications of *We* number in industrial systems.

| SYSTEM | USEFULNESS OF *We* | ENGINEERING REASON |
|---|---|---|
| Spray drying | High | Droplet deformation, atomization, and wall-deposition tendency |
| Spray nozzle | High | Liquid jet breakup and droplet formation |
| Emulsion process | High | Droplet breakup during mixing |
| Wastewater aeration | Moderate to high | Bubble deformation and gas-liquid contact |
| Bubble column | High | Bubble shape and breakup |
| Gas-liquid reactor | High | Interfacial area and mass transfer |
| Foam system | Moderate | Bubble stability and surface deformation |
| Coating process | High | Liquid film deformation and surface stability |

To show the practical use of *We*, industrial examples are presented in **Table 36**. The calculation results, industrial interpretation, and engineering handling strategies are summarized in **Table 37**. The *We* increases when fluid density, velocity, or characteristic length increases, and decreases when surface tension increases. This means that high-velocity systems, large droplets, and high-density fluids tend to have higher *We*. In contrast, systems with strong surface tension tend to resist droplet or bubble deformation.

**Table 36.** Input data for industrial *We* number examples

| CASE | INDUSTRIAL APPLICATION | ρf (kg/m³) | vc (m/s) | *db* or *Lc* (m) | σ (N/m) | *We* |
|---|---|---|---|---|---|---|
| 1 | Gentle oil-water emulsion mixing | 1000.00 | 0.05 | $1.0 \times 10^{-3}$ | 0.030 | 0.08 |
| 2 | Bubble formation in wastewater aeration | 1000.00 | 0.15 | $3.0 \times 10^{-3}$ | 0.072 | 0.94 |
| 3 | Fine droplet transport in the spray dryer | 1.00 | 60.00 | $5.0 \times 10^{-5}$ | 0.050 | 3.60 |
| 4 | High-speed atomizing gas around the spray droplet | 1.20 | 150.00 | $1.0 \times 10^{-4}$ | 0.050 | 54.00 |
| 5 | Liquid jet from process nozzle | 1000.00 | 5.00 | $1.0 \times 10^{-3}$ | 0.072 | 347.20 |
| 6 | High-pressure water spray nozzle | 1000.00 | 20.00 | $5.0 \times 10^{-4}$ | 0.072 | 2777.80 |

In spray drying, the *We* is useful for evaluating droplet breakup and deformation near the atomizer. A low *We* number indicates that droplets may remain relatively stable, while a high *We* number indicates that atomization and droplet breakup are more likely. However, excessively high *We* number may also produce very fine droplets, which can increase powder loss, wall deposition, cyclone loading, and non-uniform residence time. Atomizing gas velocity, feed flow rate, nozzle design, spray angle, and chamber airflow should be optimized.

In wastewater aeration, the *We* number can help evaluate bubble deformation and breakup. Moderate bubble deformation can improve gas-liquid contact area and oxygen transfer. However, excessive turbulence may cause unstable flow, foam formation, and energy inefficiency. Therefore, diffuser type, air flow rate, liquid depth, and aeration intensity should be selected based on the required oxygen transfer and mixing performance.

In emulsion and mixing processes, the *We* number helps determine whether mixing energy is sufficient to deform and break droplets. If the *We* number is too low, droplets may remain large, and the emulsion may be unstable. If the *We* number is sufficiently high, droplet breakup can occur, and smaller droplets may be formed. However, the *We* number should also be evaluated together with *Ca*, *Re*, and *Np* numbers because droplet breakup is affected by inertia, viscosity, surface tension, and mixing energy.

**Table 37.** Summary of *We* number results and engineering handling strategies.

| CASE | INDUSTRIAL APPLICATION | *We* VALUE | INTERFACIAL CONDITION | INDUSTRIAL INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|
| 1 | Gentle oil-water emulsion mixing | 0.083 | Surface tension dominated | Droplets tend to remain stable and are difficult to break. | Increase mixing speed, use a suitable impeller, or add an emulsifier if smaller droplets are required. |
| 2 | Bubble formation in wastewater aeration | 0.938 | Near critical deformation | Bubble deformation may begin, but strong breakup is limited. | Adjust aeration rate, diffuser design, and water depth to improve gas-liquid contact. |
| 3 | Fine droplet transport in a spray dryer | 3.600 | Moderate deformation | Droplets may deform in high-velocity drying air. | Control drying-air velocity, atomizer position, and droplet size to reduce wall deposition. |
| 4 | High-speed atomizing gas around the spray droplet | 54.0 | Strong deformation or breakup | Atomizing gas can break droplets into smaller droplets. | Optimize atomizing pressure, gas velocity, and spray angle to obtain the desired droplet size. |
| 5 | Liquid jet from process nozzle | 347.2 | Jet breakup likely | A liquid jet is unstable and likely to form droplets. | Select the proper nozzle diameter and operating pressure to control spray pattern and droplet distribution. |
| 6 | High-pressure water spray nozzle | 2777.8 | Strong atomization | Very strong spray formation and droplet breakup occur. | Check nozzle erosion, spray coverage, droplet size distribution, and pump pressure stability. |

The *We* number is a practical tool for evaluating interfacial deformation, droplet breakup, bubble deformation, spray formation, and atomization. It is especially useful for spray drying, nozzles, emulsification, wastewater aeration, and gas-liquid reactors. However, the interpretation of the *We* number should consider the process geometry, fluid properties, turbulence level, and operating condition.

### 3.1.9. Capillary Number: Viscous Effect on Interfacial Flow Systems

Capillary number, *Ca*, is used to compare the viscous force with the surface tension force in a fluid system. This dimensionless number is important in chemical engineering processes involving droplets, bubbles, emulsions, coating, liquid films, foams, porous media flow, and multiphase systems. The general interpretation of *Ca* is presented in **Table 38**. A low *Ca* indicates that surface tension is stronger than the viscous force. Under this condition, droplets or bubbles tend to maintain their shape. A high *Ca* indicates that the viscous force is strong enough to deform the interface. However, *Ca* should not be interpreted using one universal threshold only. The actual deformation condition depends on fluid viscosity, interfacial tension, velocity, droplet size, flow geometry, and turbulence level. The *Ca* is useful in many industrial systems, as summarized in **Table 39**. The *Ca* is especially useful when viscous stress affects an interface. In emulsification, it helps evaluate whether droplets can be deformed or broken by viscous flow. In coating processes, it helps evaluate whether a liquid film can spread smoothly or become unstable. In spray drying, *Ca* may be used to evaluate viscous feed droplets, especially when the feed contains polymers, starch, proteins, or suspended solids.

**Table 38.** General interpretation of *Ca* number.

| CONDITION | DOMINANT FORCE | GENERAL MEANING |
|---|---|---|
| *Ca* << 1 | Surface tension dominates | Interface tends to remain stable |
| *Ca* ≈ 1 | Viscous force and surface tension are comparable | Interface may deform significantly |
| *Ca* >> 1 | Viscous force dominates | Droplet, bubble, or film deformation becomes strong |

**Table 39.** Common applications of the *Ca* number in industrial systems.

| SYSTEM | USEFULNESS OF *Ca* | ENGINEERING REASON |
|---|---|---|
| Emulsion process | High | Droplet deformation caused by viscous stress |
| Coating process | High | Liquid-film stability and spreading behavior |
| Spray drying feed preparation | Moderate to high | Viscous deformation of droplets or feed emulsions |
| Bubble column | Moderate | Bubble deformation in viscous liquid |
| Foam system | Moderate | Bubble-film deformation and stability |
| Porous media flow | High | Displacement of one fluid by another fluid |
| Microfluidic system | High | Droplet formation and interface control |
| Wastewater sludge flow | Moderate | Interface behavior in viscous multiphase flow |

To show the practical use of the *Ca* number, industrial examples are presented in **Table 40**. The examples are arranged from low *Ca* to high *Ca*. The *Ca* number is defined as $Ca = \frac{\mu_f v_c}{\sigma}$. The calculation results, industrial interpretation, and engineering handling strategies are summarized in **Table 41**. The *Ca* increases when viscosity or velocity increases, and decreases when surface tension increases. Therefore, high-viscosity liquids and fast interfacial motion tend to produce higher *Ca*. In contrast, systems with strong surface tension tend to resist deformation and maintain a stable interface. The *Ca* is closely related to the *We* number, but both numbers describe different effects. Weber number compares the inertial force with the surface tension force, while the *Ca* number compares the viscous force with the surface tension force. Therefore, the Weber number is more useful for high-speed sprays, jets, and atomization, while the *Ca* number is more useful for viscous emulsions, coating flows, liquid films, and slow interfacial deformation.

**Table 40.** Input data for industrial *Ca* number examples.

| CASE | INDUSTRIAL APPLICATION | $\mu_f$ (Pa.s) | $v_c$ (m/s) | σ (N/m) | *Ca* |
|---|---|---|---|---|---|
| 1 | Water droplet moving slowly in the air | $1.8 \times 10^{-5}$ | 1.00 | 0.072 | 0.00025 |
| 2 | Bubble movement in wastewater aeration | $1.0 \times 10^{-3}$ | 0.10 | 0.072 | 0.00139 |
| 3 | Low-viscosity oil-water emulsion mixing | $5.0 \times 10^{-3}$ | 0.20 | 0.030 | 0.03333 |
| 4 | Spray-drying feed droplet with moderate viscosity | 0.020 | 1.00 | 0.050 | 0.40000 |
| 5 | Viscous coating liquid on the surface | 0.100 | 0.50 | 0.035 | 1.43000 |
| 6 | High-viscosity polymer solution in the mixing process | 1.000 | 0.20 | 0.040 | 5.00000 |

**Table 41.** Summary of *Ca* number results and engineering handling strategies.

| CASE | INDUSTRIAL APPLICATION | *Ca* VALUE | INTERFACIAL CONDITION | INDUSTRIAL INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|
| 1 | Water droplet moving slowly in the air | 0.00025 | Very low | Surface tension dominates, and the droplet shape is stable. | No strong viscous deformation is expected; focus on airflow and droplet size if transport is important. |
| 2 | Bubble movement in wastewater aeration | 0.00139 | Low | Bubble shape is mainly controlled by surface tension. | Adjust diffuser type, aeration rate, and water depth if smaller bubbles or better oxygen transfer are required. |
| 3 | Low-viscosity oil-water emulsion mixing | 0.03333 | Low to moderate | Viscous force begins to deform droplets, but surface tension still resists breakup. | Increase mixing speed, select a suitable impeller, or add an emulsifier if smaller droplets are required. |
| 4 | Spray-drying feed droplet with moderate viscosity | 0.40000 | Moderate | Viscous force affects droplet deformation and atomization behavior. | Control feed viscosity, feed temperature, atomizer pressure, and solid concentration. |
| 5 | Viscous coating liquid on the surface | 1.43000 | High | Viscosity strongly affects liquid-film spreading and surface stability. | Optimize coating speed, liquid viscosity, surface tension, and film thickness. |
| 6 | High-viscosity polymer solution in the mixing process | 5.00000 | Very high | Viscous force dominates; interfacial deformation is strong, but mixing may require high energy. | Use an appropriate impeller for viscous fluids, reduce viscosity if possible, increase mixing time, and check motor capacity. |

In spray drying, the *Ca* number can help evaluate the effect of feed viscosity on droplet deformation and atomization. A feed with high viscosity may produce a higher *Ca* number, indicating a stronger viscous influence. However, high viscosity can also make atomization more difficult because the liquid resists stretching and breakup. Therefore, feed viscosity, solid concentration, feed temperature, atomizer speed, atomizing gas pressure, and nozzle condition should be controlled carefully.

In emulsion processes, the *Ca* number helps determine whether viscous stress is sufficient to deform droplets against surface tension. If *Ca* is too low, droplets may remain large, and emulsion formation may be poor. If *Ca* is high enough, droplet deformation becomes easier, and smaller droplets may be formed. However, excessive viscosity may increase energy consumption and make mixing difficult.

In coating processes, the *Ca* number is useful for evaluating the balance between viscous spreading and surface tension. A low *Ca* number may produce a stable interface but slow spreading. A high *Ca* number may improve spreading, but it can also cause film instability, uneven coating thickness, or defects if the process is not controlled.

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The *Ca* number is a practical tool for evaluating viscous effects on interfaces in emulsions, coatings, spray drying feed, bubble systems, foam systems, and multiphase flow. However, it should be interpreted together with *We*, *Re*, and *Np* numbers because interfacial behavior is influenced by inertia, viscosity, surface tension, and mixing energy.

### 3.1.10. Cavitation Number: Cavitation Risk in Pumps, Valves, and Nozzles

Cavitation number, σc, is used to evaluate the possibility of cavitation in liquid-flow systems. The σc occurs when the local pressure of a liquid decreases close to or below its vapor pressure. Under this condition, vapor bubbles may form inside the liquid. When these bubbles move to a higher-pressure region, they can collapse violently and cause noise, vibration, erosion, loss of pump performance, and equipment damage. The σc is commonly used in chemical engineering systems such as pumps, valves, nozzles, pipelines, hydraulic equipment, spray systems, and process units with high liquid velocity or strong pressure reduction. This number is defined as $\frac{P_{loc}-P_v}{0.5\rho_l v_p^2}$. The numerator, $P_{loc}$ - *Pv*, represents the pressure margin above the vapor pressure. The denominator, $0.5.\rho l.vc^2$, represents the dynamic pressure of the flowing liquid. Therefore, the σc compares the available pressure margin with the kinetic energy of the liquid flow. The general interpretation of the σc is presented in **Table 42**. A higher σc generally indicates safer operation because the local pressure is sufficiently higher than the vapor pressure. A lower σc indicates that the liquid is closer to vaporization, and cavitation may occur. However, the σc does not have one universal safety limit for all equipment. The actual cavitation limit depends on pump design, valve geometry, nozzle shape, liquid temperature, vapor pressure, dissolved gas, turbulence, and operating conditions.

**Table 42.** General interpretation of cavitation number.

| CONDITION | CAVITATION RISK | GENERAL MEANING |
|---|---|---|
| High σc | Low cavitation risk | Local pressure is much higher than the vapor pressure |
| Moderate σc | Possible cavitation risk | Pressure margin is reduced and should be checked |
| Low σc | High cavitation risk | Local pressure is close to the vapor pressure |
| Very low σc | Severe cavitation risk | Vapor bubble formation is likely |

In addition, the vapor pressure, *Pv*, is not a constant value for all liquids. It depends on the type of liquid and its operating temperature. For water-based systems, such as cooling water, wastewater, and dilute slurry, the vapor pressure of water can be used as an initial approximation. However, for organic solvents or other process liquids, *Pv* should be selected from the physical property data of the corresponding material at the actual operating temperature. Since vapor pressure generally increases with temperature, hotter liquids have a smaller pressure margin ($P_{loc}$ - *Pv*) and are more prone to cavitation. Therefore, the correct selection of *Pv* is important for evaluating cavitation risk in pumps, valves, nozzles, and other liquid-flow equipment (see **Table 43**). Important note, in the table, N/A means the liquid's vapor pressure is not recommended for use because the temperature is approaching or exceeding the critical temperature, is in a supercritical state, or the chemical condition may decompose. For cavitation analysis, *Pv* is used when the material is still in the liquid phase. Vapor pressure itself is the vapor pressure/saturation pressure that depends on temperature.

**Table 43.** Approximate vapor pressure values of selected chemicals at different temperatures.

| NO | CHEMICAL | *Pv* (kPa) | | | |
|---|---|---|---|---|---|
| | | 25°C | 50°C | 100°C | 200°C |
| 1 | Water | 3.17 | 12.35 | 101.3 | 1,550 |
| 2 | Methanol | 16.9 | 55-57 | ±330 | ±3,000-4,000 |
| 3 | Ethanol | 7.9 | ±30 | ±220 | ±3,000 |
| 4 | Isopropanol | ±6 | ±22-26 | ±180-190 | ±3,000 |
| 5 | n-Propanol | ±3 | ±13-14 | ±100-110 | ±2,000 |
| 6 | n-Butanol | ±0.9 | ±4-6 | ±50-55 | ±1,000 |
| 7 | Acetone | ±30 | ±80 | ±390 | ±3,000 |
| 8 | Methyl ethyl ketone | ±12 | ±35-40 | ±180 | ±1,500 |
| 9 | Acetonitrile | ±12 | ±34 | ±175 | ±1,500-2,000 |
| 10 | Ethyl acetate | ±13 | ±40 | ±200 | ±1,500-2,000 |
| 11 | Diethyl ether | ±70 | ±160 | ±600 | N/A |
| 12 | Dichloromethane | ±58 | ±140 | ±580 | ±4,000 |
| 13 | Chloroform | ±26 | ±70 | ±300 | ±2,000 |
| 14 | Carbon tetrachloride | ±15 | ±40 | ±190 | ±1,500 |
| 15 | Benzene | ±13 | ±36-38 | ±175 | ±1,500 |
| 16 | Toluene | ±3.8 | ±12-14 | ±70-75 | ±700 |
| 17 | o-Xylene | ±1 | ±4-5 | ±30 | ±300-400 |
| 18 | n-Hexane | ±20 | ±55 | ±240 | ±1,700 |
| 19 | n-Heptane | ±6-8 | ±20-22 | ±100 | ±900 |
| 20 | n-Octane | ±2 | ±8-9 | ±50 | ±500 |
| 21 | Cyclohexane | ±13 | ±35-38 | ±170 | ±1,300 |
| 22 | Acetic acid | ±2 | ±8-10 | ±55-70 | ±300-400 |
| 23 | Formic acid | ±5-6 | ±20-30 | ±100 | ±400-500 |
| 24 | Ethylene glycol | very low, ±0.01 | ±0.1-0.2 | ±1-3 | ±100 |
| 25 | Propylene glycol | very low, ±0.01 | ±0.1-0.2 | ±2-4 | ±100-150 |
| 26 | Glycerol | extremely low | extremely low | ±0.01-0.03 | ±1-5 |
| 27 | Dimethyl sulfoxide, DMSO | ±0.06 | ±0.3 | ±4 | ±100 |
| 28 | Dimethylformamide, DMF | ±0.4 | ±1-2 | ±15 | ±300-400 |
| 29 | Aniline | ±0.1-0.2 | ±0.8 | ±7-8 | ±100-200 |
| 30 | Chlorobenzene | ±1.6 | ±7 | ±40 | ±400-500 |

Note: The values were adapted from physical-property references such as the NIST Chemistry WebBook, steam tables, DIPPR, Perry's Handbook, Aspen, REFPROP, and SDS/material databases. N/A indicates that the material may not exist as a stable liquid at the stated temperature, may exceed its critical temperature, may be in a supercritical state, or may undergo thermal degradation.

The σc is useful in different industrial systems, as summarized in **Table 44**. The σc is mainly applied to liquid systems where local pressure can drop significantly. In pump systems, a low σc may occur near the pump inlet or impeller eye. In valves and nozzles, cavitation may occur when liquid accelerates through a narrow passage and the local pressure decreases. In wastewater pumping systems, cavitation can damage pump components and reduce flow performance.

To show the practical use of σc, industrial examples are presented in **Table 45**. The examples are arranged from high to low σc, representing a transition from safer operation to higher cavitation risk.

**Table 44.** Common applications of cavitation number in industrial systems.

| SYSTEM | USEFULNESS OF σc | ENGINEERING REASON |
|---|---|---|
| Centrifugal pump | High | To evaluate cavitation risk at the pump inlet or impeller eye |
| Control valve | High | To evaluate cavitation due to pressure reduction across the valve |
| Nozzle | High | To evaluate vapor formation under high-velocity liquid flow |
| Spray system | Moderate to high | To evaluate pressure reduction and atomization-related cavitation |
| Pipeline restriction | High | To evaluate cavitation near sudden contraction or orifice |
| Heat exchanger liquid line | Moderate | To evaluate cavitation risk if the pressure drops significantly |
| Boiler feedwater system | High | To avoid vapor formation in a hot, pressurized liquid |
| Wastewater pumping system | Moderate to high | To avoid pump noise, vibration, and impeller erosion |

**Table 45.** Input data for industrial cavitation number examples.

| CASE | INDUSTRIAL APPLICATION | $P_{loc}$ (Pa) | $P_v$ (Pa) | $\rho l$ (kg/m³) | $vc$ (m/s) | σc |
|---|---|---|---|---|---|---|
| 1 | Cooling water pipe at low velocity | 300,000 | 3,000 | 1,000 | 2.00 | 148.50 |
| 2 | Pump the suction line with sufficient pressure | 200,000 | 12,000 | 1,000 | 3.00 | 41.80 |
| 3 | Wastewater pump inlet | 120,000 | 3,000 | 1,000 | 4.00 | 14.60 |
| 4 | Control valve with pressure reduction | 80,000 | 12,000 | 1,000 | 6.00 | 3.78 |
| 5 | Hot water pump suction line | 60,000 | 20,000 | 980 | 7.00 | 1.67 |
| 6 | High-velocity liquid nozzle | 40,000 | 12,000 | 1,000 | 10.00 | 0.56 |

The calculation results, industrial interpretation, and engineering handling strategies are summarized in **Table 46**. The σc decreases when local pressure decreases, vapor pressure increases, or liquid velocity increases. Therefore, cavitation is more likely in high-velocity regions, low-pressure zones, and hot liquid systems. Hot liquids are more sensitive to cavitation because vapor pressure increases with temperature.

In pump systems, cavitation can occur when the suction pressure is too low. This may happen due to insufficient liquid level, long suction pipe, excessive fittings, clogged strainer, high liquid temperature, or high pump speed. Engineering strategies include increasing suction pressure, reducing suction pipe losses, lowering liquid temperature, cleaning strainers, increasing liquid level, and checking the Net Positive Suction Head requirement.

In control valves and nozzles, cavitation may occur because liquid velocity increases through a restricted area and static pressure decreases. If the local pressure falls close to the vapor pressure, vapor bubbles may form. When the pressure recovers downstream, these bubbles may collapse and damage the valve, nozzle, or pipe wall. Engineering strategies include selecting the proper valve type and size, avoiding excessive throttling, using multi-stage pressure reduction, increasing downstream pressure, and using cavitation-resistant materials.

In wastewater pumping systems, cavitation can cause pump vibration, noise, reduced flow rate, impeller erosion, and poor operational reliability. Wastewater systems may also experience additional problems due to blockage, sludge accumulation, inlet screen fouling, and air entrainment. Therefore, pump inlet conditions, suction pressure, liquid level, and inlet screen cleanliness should be monitored.

**Table 46.** Summary of cavitation number results and engineering handling strategies.

| CASE | INDUSTRIAL APPLICATION | σc | CAVITATION RISK | INDUSTRIAL INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|
| 1 | Cooling water pipe at low velocity | 148.50 | Very low | Local pressure is much higher than vapor pressure. | Maintain normal operating pressure and monitor pressure drop. |
| 2 | Pump suction line with sufficient pressure | 41.80 | Low | Pump suction condition is generally safe. | Keep the suction line short, reduce unnecessary fittings, and maintain an adequate liquid level. |
| 3 | Wastewater pump inlet | 14.60 | Low to moderate | Cavitation is not dominant, but the suction condition should be checked. | Clean inlet screen, avoid blockage, maintain submergence, and check pump noise. |
| 4 | Control valve with pressure reduction | 3.78 | Moderate | Pressure reduction across the valve may increase cavitation tendency. | Use proper valve sizing, avoid excessive throttling, and check downstream pressure. |
| 5 | Hot water pump suction line | 1.67 | High | High liquid temperature increases vapor pressure and reduces pressure margin. | Reduce liquid temperature if possible, increase suction pressure, reduce suction velocity, and check NPSH requirement. |
| 6 | High-velocity liquid nozzle | 0.56 | Very high | High velocity and low local pressure may cause vapor bubble formation. | Reduce nozzle velocity, increase upstream pressure, redesign nozzle geometry, or use cavitation-resistant material. |

The σc is a practical tool for evaluating cavitation tendency in pumps, valves, nozzles, pipelines, and liquid-handling equipment. A low σc indicates that the liquid pressure is close to the vapor pressure, and cavitation may occur. However, σc should be interpreted together with equipment design, flow geometry, liquid temperature, vapor pressure, pressure drop, and NPSH information.

### 3.2. Integrated Case Studies

The selection of relevant dimensionless numbers can be performed using simple yes-or-no questions. If the answer is yes, the related dimensionless number should be considered. The step-by-step selection guide is shown in **Table 47**.

After the relevant dimensionless numbers are selected using **Table 47**, each case study can be solved by listing the process zones, selecting the appropriate variables, calculating the selected dimensionless numbers, interpreting the results, and proposing handling strategies.

The integrated case studies discussed are arranged from the simplest case to the more complex case, as summarized in **Table 48**. **Figures 2-7** illustrate representative chemical engineering systems used. These systems were selected because they involve different combinations of fluid-flow phenomena, such as liquid flow, gas flow, pressure drop, mixing, particle transport, interfacial deformation, aeration, drying, and separation. Each system requires more than one dimensionless number to describe its behavior; therefore, the figure provides a visual overview of how the selected dimensionless numbers can be applied together in practical equipment analysis.

**Table 47.** Step-by-step guide for selecting relevant dimensionless numbers.

| NO | SIMPLE QUESTION | IF YES, USE | MAIN PURPOSE |
|---|---|---|---|
| 1 | Does the system have fluid flow? | *Re* | To identify laminar, transitional, or turbulent flow |
| 2 | Does the system have a gas flow? | *Ma* | To evaluate gas compressibility |
| 3 | Does gravity or free surface affect the flow? | *Fr* | To compare inertia and gravity |
| 4 | Does the system have a pressure drop? | *Eu* | To evaluate pressure loss or hydraulic resistance |
| 5 | Does the system use an impeller or a mixer? | *Np* | To evaluate the mixing power demand |
| 6 | Does the fluid carry particles, droplets, flocs, or solids? | *Stk* | To evaluate particle response to fluid motion |
| 7 | Are droplets, bubbles, sprays, or jets formed or broken? | *We* | To evaluate interfacial deformation by inertia |
| 8 | Is viscous interfacial deformation important? | *Ca* | To evaluate interfacial deformation by viscosity |
| 9 | Can liquid vapor bubbles form due to low pressure? | σc | To evaluate cavitation risk |

**Table 48.** Integrated case studies arranged from simpler to more complex applications.

| NO | INTEGRATED CASE STUDY | MAIN PROCESS ZONES DISCUSSED | RELEVANT DIMENSIONLESS |
|---|---|---|---|
| 1 | Heat exchanger system | Cooling-water tube side, tube-side pressure drop, suspended particles or fouling, cooling-water pump, hot gas shell side, and gas-side pressure drop | *Re, Ma, Eu, Stk, σc* |
| 2 | Slurry pipeline, pump, valve, and nozzle system | Slurry pipe, pump discharge, pump suction, control valve, particle transport, nozzle discharge, discharge into tank, and mixing tank before pumping | *Re, Fr, Eu, Np, Stk, We, σc* |
| 3 | Mixing vessel | Impeller, baffle, vortex, particle suspension, emulsion or droplet breakup, recirculation line, and cavitation near the impeller | *Re, Fr, Eu, Np, Stk, We, Ca, σc* |
| 4 | Wastewater aeration basin | Inlet channel, aeration basin, mechanical mixer, bubble formation, sludge floc, diffuser pressure drop, and pump cavitation | *Re, Fr, Eu, Np, Stk, We, Ca, σc* |
| 5 | Aerated fermentation bioreactor | Bulk liquid broth, impeller region, free surface, sparger, air bubbles, cell or floc suspension, air line, and cavitation near impeller | *Re, Ma, Fr, Eu, Np, Stk, We, Ca, σc* |
| 6 | Spray-drying process | Feed tank, feed pipe, atomizer, drying chamber, wall region, cyclone, and exhaust duct | *Re, Ma, Fr, Eu, Np, Stk, We, Ca, σc* |

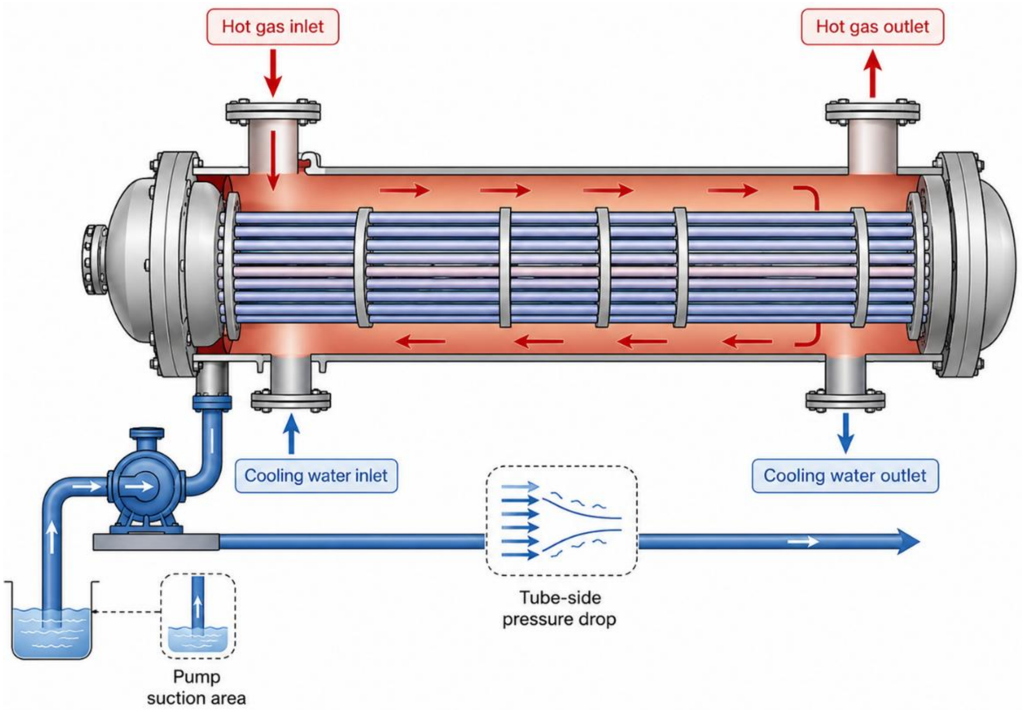


**Figure 2.** Schematic illustration of shell-and-tube heat exchanger system. This system involves different combinations of operating parameters and fluid-flow phenomena, allowing several dimensionless numbers to be calculated and interpreted together for process analysis and engineering decision-making.

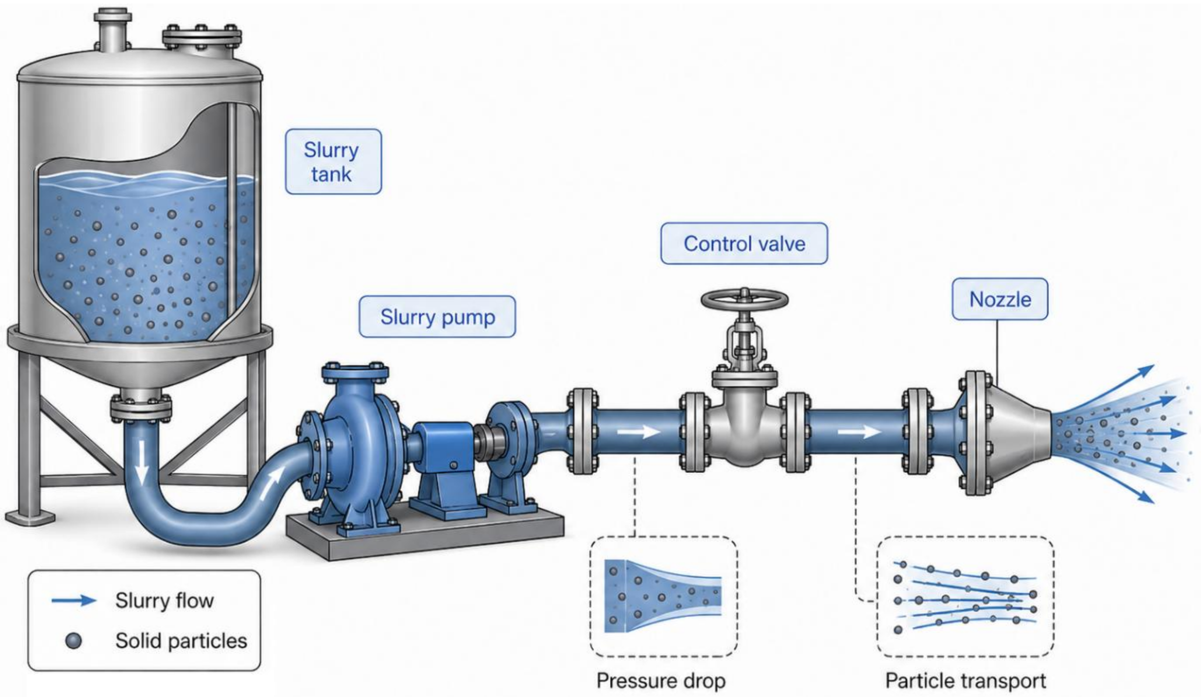


**Figure 3.** Schematic illustration of slurry pipeline-pump-valve-nozzle system. This system involves different combinations of operating parameters and fluid-flow phenomena, allowing several dimensionless numbers to be calculated and interpreted together for process analysis and engineering decision-making.

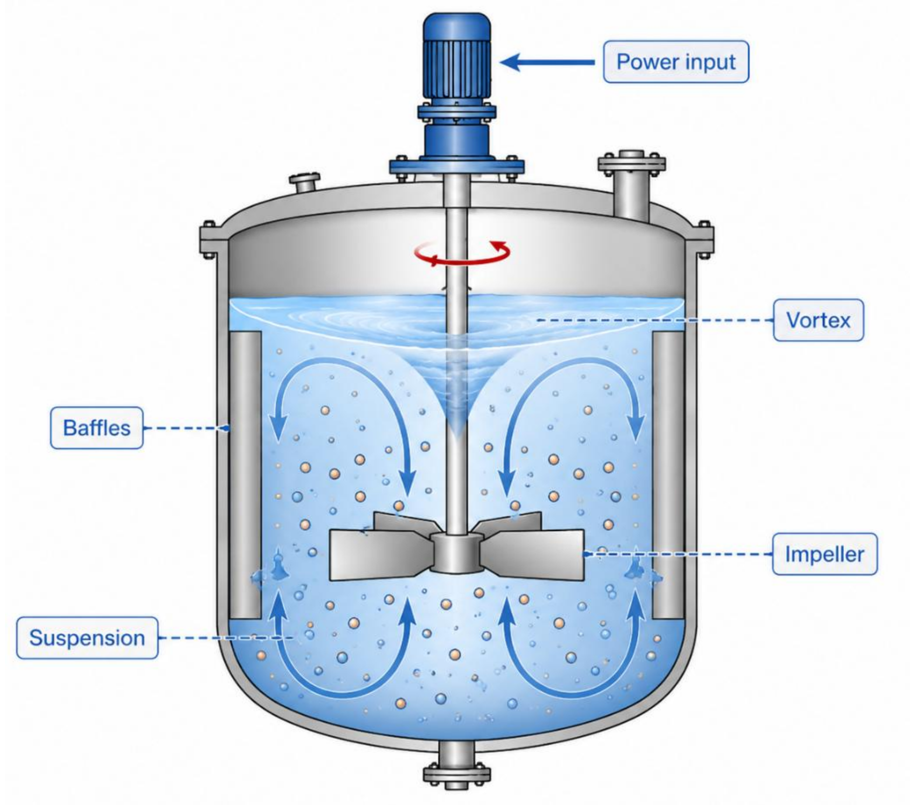


**Figure 4.** Schematic illustration of a stirred mixing vessel. This system involves different combinations of operating parameters and fluid-flow phenomena, allowing several dimensionless numbers to be calculated and interpreted together for process analysis and engineering decision-making.

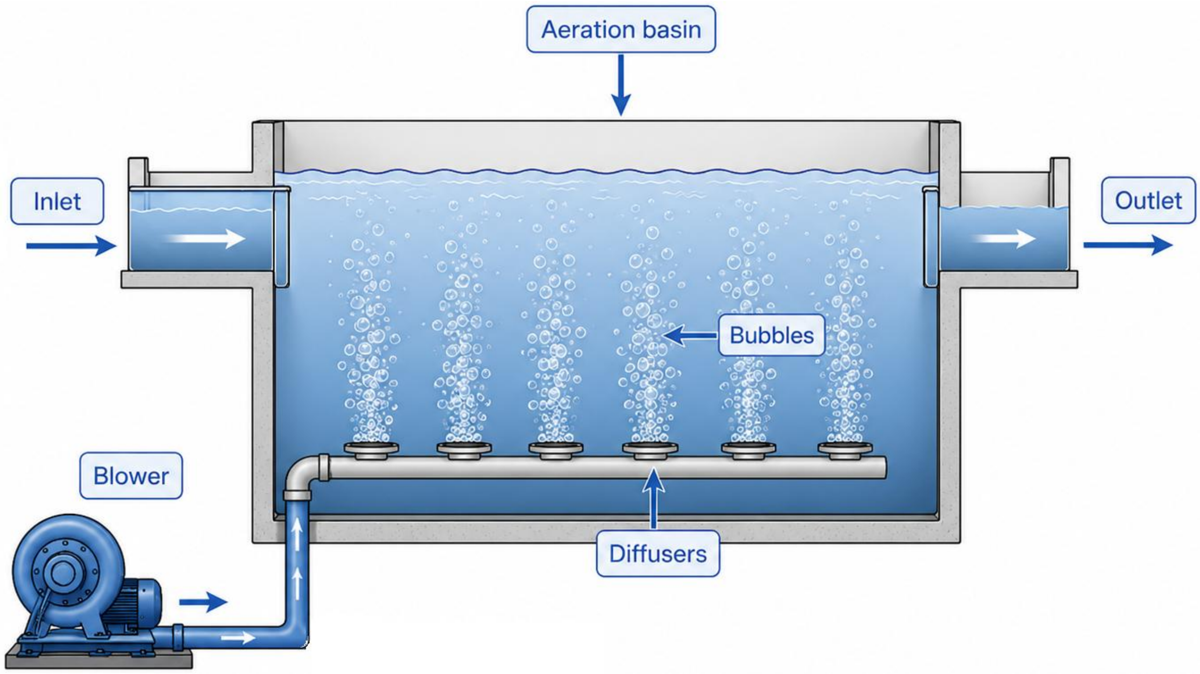


**Figure 5.** Schematic illustration of a wastewater aeration basin. This system involves different combinations of operating parameters and fluid-flow phenomena, allowing several dimensionless numbers to be calculated and interpreted together for process analysis and engineering decision-making.

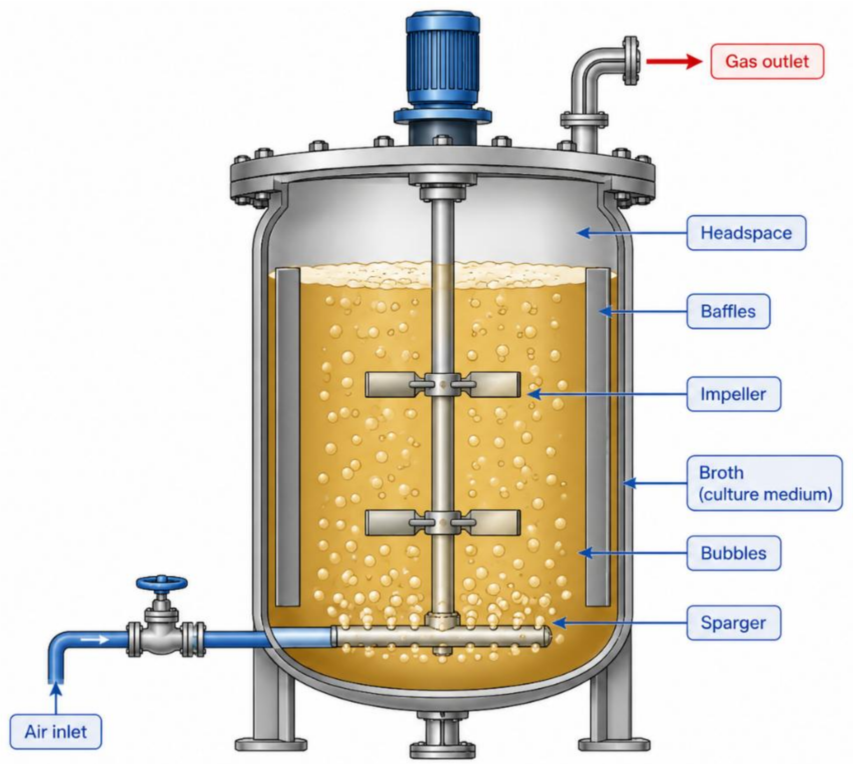


**Figure 6.** Schematic illustration of an aerated fermentation bioreactor. This system involves different combinations of operating parameters and fluid-flow phenomena, allowing several dimensionless numbers to be calculated and interpreted together for process analysis and engineering decision-making.

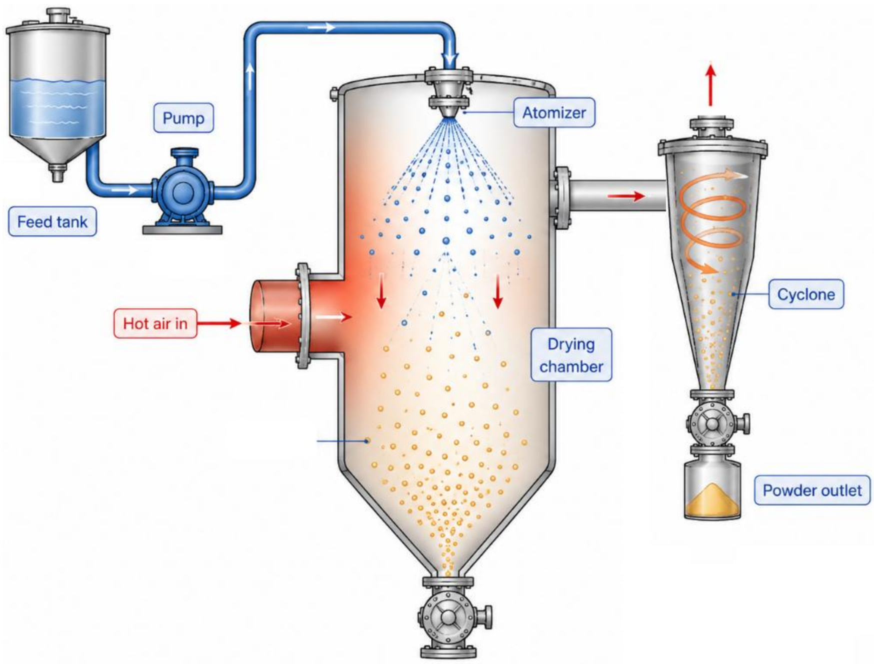


**Figure 7.** Schematic illustration of a spray drying system with a cyclone separator. This system involves different combinations of operating parameters and fluid-flow phenomena, allowing several dimensionless numbers to be calculated and interpreted together for process analysis and engineering decision-making.

### 3.2.1. Integrated Case Study: Heat Exchanger System

A shell-and-tube heat exchanger is used to cool hot process gas using cooling water. Cooling water flows through the tube side, while hot gas flows through the shell side. The operating data are presented in **Table 49**. The step-by-step selection, calculation, interpretation, and handling strategy are shown in **Table 50**. The relevant dimensionless numbers for the heat exchanger case are *Re*, *Ma*, *Eu*, *Stk*, and $\sigma c$. The main engineering concerns are tube-side pressure drop, fouling control, pump suction condition, and gas-side deposit buildup.

**Table 49.** Operating data for the heat exchanger case.

| NO | PARAMETER | SYMBOL | VALUE | UNIT |
|---|---|---|---|---|
| 1 | Cooling-water density | $\rho l$ | 1000 | kg/m³ |
| 2 | Cooling-water viscosity | $\mu f$ | $1.0 \times 10^{-3}$ | Pa.s |
| 3 | Cooling-water velocity in the tube | $Vl$ | 1.50 | m/s |
| 4 | Tube inside diameter | $Dp$ | 0.025 | M |
| 5 | Tube-side pressure drop | $\Delta P_l$ | 12000 | Pa |
| 6 | Local pressure at the pump suction | $P_{local}$ | 150000 | Pa |
| 7 | Vapor pressure of cooling water | $Pv$ | 3000 | Pa |
| 8 | Pump-line liquid velocity | $Vp$ | 3.00 | m/s |
| 9 | Suspended-particle density | $\rho p$ | 2650 | kg/m³ |
| 10 | Suspended-particle diameter | $Dp$ | $5.0 \times 10^{-5}$ | m |
| 11 | Characteristic length near the tube wall | $Lc$ | 0.025 | m |
| 12 | Characteristic water velocity | $Vc$ | 1.50 | m/s |
| 13 | Hot gas density | $\rho g$ | 0.80 | kg/m³ |
| 14 | Hot gas velocity | $Vg$ | 40.00 | m/s |
| 15 | Speed of sound in hot gas | $C$ | 430 | m/s |
| 16 | Gas-side pressure drop | $\Delta P_g$ | 800 | Pa |

**Table 50.** Step-by-step selection, calculation, interpretation, and handling strategy for the heat exchanger case.

| STEP NO | QUESTION | ANSWER AND DECISION | HOW TO SOLVE | FORMULA AND CALCULATION | INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|---|
| 1 | Does the system have fluid flow? | Yes. Cooling water flows on the tube side. Use *Re*. | Calculate Re for cooling-water tube flow. $Re = \frac{\rho v D_p}{\mu}$ | *Re* = 1000 × 1.50 × 0.025 / 1.0 × $10^{-3}$ = 37500 | Cooling-water flow is turbulent. | Turbulent flow is good for heat transfer, but pressure drop and possible erosion should be monitored. |
| 2 | Does the system have a gas flow? | Yes. Hot gas flows on the shell side. Use *Ma*. | Calculate Ma for hot gas flow. $Ma = \frac{v_g}{c_T}$ | *Ma* = 40.00 / 430 = 0.093 | Gas compressibility is not dominant because *Ma* < 0.3. | Nearly incompressible gas-flow assumption may be acceptable for preliminary hydraulic analysis. |
| 3 | Does gravity or free surface strongly affect the flow? | No. The heat exchanger is a closed-flow system. *Fr* is not required. | No calculation is needed. $Fr = \frac{v}{\sqrt{gL}}$ | Not calculated. | No open-channel flow, free surface, or vortex behavior is considered. | No Fr-based action is required. |
| 4 | Does the system have a pressure drop? | Yes. Pressure drop occurs on the tube side. Use $Eu_l$. | Calculate Eu for tube-side liquid flow. $Eu_l = \frac{\Delta P_l}{\rho_l v_l^2}$. | $Eu_l$ = 12000 / (1000 × $1.50^2$) = 5.33 | Tube-side pressure drop is significant. | Check pump capacity, tube cleanliness, fouling, scaling, and possible flow restriction. |
| 5 | Does the system also have a gas-side pressure drop? | Yes. Pressure drop occurs on the gas side. Use $Eu_g$. | Calculate Eu for gas-side flow. $Eu_g = \frac{\Delta P_g}{\rho_g v_g^2}$. | $Eu_g$ = 800 / (0.80 × $40.00^2$) = 0.625 | Gas-side pressure drop is moderate. | Check blower capacity and monitor dust, deposit buildup, or fouling on the gas side. |
| 6 | Does the system use an impeller or a mechanical mixer? | No. There is no impeller in the heat exchanger. *Np* is not required. | No calculation is needed. $Np = \frac{P_w}{\rho_{mix} N^3 D_{imp}^5}$ | Not calculated. | Mechanical agitation is not present. | No *Np*-based action is required. |

**Table 50 (continue).** Step-by-step selection, calculation, interpretation, and handling strategy for the heat exchanger case.

| STEP NO | QUESTION | ANSWER AND DECISION | HOW TO SOLVE | FORMULA AND CALCULATION | INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|---|
| 7 | Does the fluid carry particles, solids, or fouling material? | Yes. Cooling water may contain suspended particles. Use *Stk*. | Calculate $\tau_p = \frac{\rho_p d_p^2}{18\mu_f}$; $\tau_f = \frac{L_c}{v_c}$; and $Stk = \frac{\tau_p}{\tau_f}$. | $\tau p = 2650 \times (5.0 \times 10^{-5})^2 / (18 \times 1.0 \times 10^{-3}) = 3.68 \times 10^{-4}$ s; $\tau f = 0.025 / 1.50 = 0.0167$ s; *Stk* = $3.68 \times 10^{-4} / 0.0167 = 0.0221$ | Fine particles generally follow the cooling-water flow. However, fouling may still occur due to adhesion, scaling, biofilm, or low-velocity zones. | Monitor pressure drop over time, clean tubes periodically, control water quality, and inspect for scaling or biological fouling. |
| 8 | Are droplets, bubbles, sprays, or jets formed or broken? | No. No droplet, bubble, spray, or jet breakup is considered. *We* is not required. | No calculation is needed. General: $We = \frac{\rho_f v_c^2 L_C}{\sigma}$ Droplet: $We = \frac{\rho_f v_c^2 d_b}{\sigma}$ | Not calculated. | Interfacial breakup is not part of this heat exchanger case. | No *We*-based action is required. |
| 9 | Is viscous interfacial deformation important? | No. No emulsion, coating, liquid film, or bubble-interface deformation is analyzed. *Ca* is not required. | No calculation is needed. $Ca = \frac{\mu_f v_c}{\sigma}$ | Not calculated. | Viscous interfacial deformation is not dominant. | No *Ca*-based action is required. |
| 10 | Can liquid vapor bubbles form due to low pressure? | Yes. Cooling water is supplied by a pump. Use σc. | Calculate σc near the cooling-water pump suction. $\sigma_c = \frac{P_{loc} - P_v}{0.5\rho_l v_p^2}$ | $\sigma c = (150000 - 3000) / (0.5 \times 1000 \times 3.00^2) = 32.7$ | Cavitation risk is low under the assumed condition. | Maintain suction pressure, clean strainers, avoid excessive water temperature, and prevent suction-line blockage. |

### 3.2.2. Integrated Case Study: Slurry Pipeline, Pump, Valve, and Nozzle System

A slurry pipeline system is used to transport a solid-liquid mixture from a mixing tank through a pump, valve, pipeline, and nozzle. The operating data are presented in **Table 51**.

**Table 51.** Operating data for the slurry pipeline case.

| NO | PARAMETER | SYMBOL | VALUE | UNIT |
|---|---|---|---|---|
| 1 | Slurry density | $\rho m$ | 1200 | kg/m³ |
| 2 | Slurry viscosity | $\mu f$ | 0.020 | Pa.s |
| 3 | Pipe diameter | *Dp* | 0.050 | m |
| 4 | Slurry velocity in a pipe | *vl* | 1.20 | m/s |
| 5 | Pressure drop in the pipeline | $\Delta P$ | 18000 | Pa |
| 6 | Local pressure at the pump suction | $P_{loc}$ | 90000 | Pa |
| 7 | Vapor pressure of the liquid phase | *Pv* | 3000 | Pa |
| 8 | Velocity near the pump suction | *vp* | 3.00 | m/s |
| 9 | Solid particle density | $\rho p$ | 2650 | kg/m³ |
| 10 | Solid particle diameter | *Dp* | $1.5 \times 10^{-4}$ | m |
| 11 | Characteristic length for particle response | *Lc* | 0.050 | m |
| 12 | Characteristic slurry velocity | *vc* | 1.20 | m/s |
| 13 | Nozzle diameter | *dn* | 0.010 | m |
| 14 | Nozzle exit velocity | *vn* | 8.00 | m/s |
| 15 | Surface tension | *sigma* | 0.072 | N/m |
| 16 | Discharge length scale | *Ld* | 0.20 | m |
| 17 | Mixing power input | *Pw* | 600 | W |
| 18 | Impeller speed | *N* | 4.00 | 1/s |
| 19 | Impeller diameter | $D_{imp}$ | 0.40 | m |
| 20 | Gravitational acceleration | *g* | 9.81 | m/s² |

The step-by-step selection, calculation, interpretation, and handling strategy are shown in **Table 52**. The relevant dimensionless numbers for the slurry pipeline case are *Re*, *Fr*, *Eu*, *Np*, *Stk*, *We*, and $\sigma$c. The main engineering concerns are slurry transport, pressure drop, particle settling, pump cavitation, nozzle jet behavior, and mixing quality before pumping.

**Table 52.** Step-by-step selection, calculation, interpretation, and handling strategy for the slurry pipeline case.

| STEP NO | QUESTION | ANSWER AND DECISION | HOW TO SOLVE | FORMULA AND CALCULATION | INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|---|
| 1 | Does the system have fluid flow? | Yes. Slurry flows through the pipeline. Use *Re*. | Calculate *Re* for slurry pipe flow. | $Re = 1200 \times 1.20 \times 0.050 / 0.020 = 3600$ | Slurry flow is transitional to turbulent. | Maintain velocity above the settling limit and avoid sudden flow reduction. |
| 2 | Does the system have a gas flow? | No. This case focuses on liquid slurry flow. *Ma* is not required. | No calculation is needed. | Not calculated. | Gas compressibility is not involved. | No *Ma*-based action is required. |
| 3 | Does gravity or free surface strongly affect the flow? | Yes. Slurry is discharged into a tank through a nozzle. Use *Fr*. | Calculate *Fr* for the discharge region. | $Fr = 8.00 / \text{sqrt}(9.81 \times 0.20) = 5.71$ | Jet inertia dominates over gravity. | Install diffuser, reduce jet velocity, or adjust discharge angle to reduce splashing and erosion. |
| 4 | Does the system have a pressure drop? | Yes. Pressure drop occurs in the slurry pipeline. Use *Eu*. | Calculate *Eu* for slurry pipe flow. | $Eu = 18000 / (1200 \times 1.20^2) = 10.4$ | Pipeline pressure drop is significant. | Check pump capacity, pipe diameter, fittings, valve opening, and possible sediment buildup. |
| 5 | Does the system use an impeller or a mechanical mixer? | Yes. The slurry is mixed before pumping. Use *Np*. | Calculate *Np* for the mixing tank. | $Np = 600 / (1200 \times 4.00^3 \times 0.40^5) = 0.763$ | Mixing power demand is moderate to relatively high. | Check solid suspension in the tank and use baffles or a suitable impeller if dead zones occur. |
| 6 | Does the fluid carry particles, solids, or fouling material? | Yes. The slurry contains solid particles. Use *Stk*. | Calculate $\tau p$, $\tau f$, then *Stk*. | $\tau p = 2650 \times (1.5 \times 10^{-4})^2 / (18 \times 0.020) = 1.66 \times 10^{-4}$ s; $\tau f = 0.050 / 1.20 = 0.0417$ s; $Stk = 1.66 \times 10^{-4} / 0.0417 = 0.00398$ | Fine particles generally follow the slurry flow. However, coarse particles may still settle. | Check particle-size distribution, maintain sufficient velocity, and avoid low-velocity zones. |

**Table 52 (continue).** Step-by-step selection, calculation, interpretation, and handling strategy for the slurry pipeline case.

| STEP NO | QUESTION | ANSWER AND DECISION | HOW TO SOLVE | FORMULA AND CALCULATION | INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|---|
| 7 | Are droplets, bubbles, sprays, or jets formed or broken? | Yes. The slurry exits through a nozzle as a jet. Use *We*. | Calculate *We* for nozzle discharge. | $We = 1200 \times 8.00^2 \times 0.010 / 0.072 = 10667$ | The nozzle jet is strongly inertia-dominated. | Use proper nozzle design, reduce excessive velocity, and protect surfaces from erosion or splashing. |
| 8 | Is viscous interfacial deformation important? | No. This case does not focus on emulsion, coating, or viscous interface deformation. *Ca* is not required. | No calculation is needed. | Not calculated. | Viscous interfacial deformation is not dominant. | No *Ca*-based action is required. |
| 9 | Can liquid vapor bubbles form due to low pressure? | Yes. The slurry is transported by a pump. Use $\sigma c$. | Calculate $\sigma c$ near the pump suction. | $\sigma c = (90000 - 3000) / (0.5 \times 1200 \times 3.00^2) = 16.1$ | Cavitation risk is relatively low under the assumed condition. | Maintain suction pressure, clean the suction line, avoid air entrainment, and keep the tank level sufficient. |

### 3.2.3. Integrated Case Study: Mixing Vessel

A mixing vessel is used to prepare a slurry or liquid feed before further processing. The system uses an impeller and baffles to improve circulation, reduce vortex formation, and maintain particle suspension. The operating data are presented in **Table 53**.

**Table 53.** Operating data for the mixing vessel case.

| NO | PARAMETER | SYMBOL | VALUE | UNIT |
|---|---|---|---|---|
| 1 | Mixed-fluid density | $\rho m$ | 1100 | kg/m³ |
| 2 | Mixed-fluid viscosity | $\mu f$ | 0.020 | Pa.s |
| 3 | Surface tension | *sigma* | 0.050 | N/m |
| 4 | Tank diameter | *Dt* | 1.00 | m |
| 5 | Liquid height | *H* | 1.20 | m |
| 6 | Impeller diameter | $D_{imp}$ | 0.30 | m |
| 7 | Impeller rotational speed | *N* | 5.00 | 1/s |
| 8 | Impeller tip velocity | *vtip* | 4.71 | m/s |
| 9 | Characteristic circulation velocity | *vc* | 1.50 | m/s |
| 10 | Mixing power input | *Pw* | 180 | W |
| 11 | Particle density | $\rho p$ | 2650 | kg/m³ |
| 12 | Particle diameter | *Dp* | $1.0 \times 10^{-4}$ | m |
| 13 | Droplet diameter | *db* | $5.0 \times 10^{-4}$ | m |
| 14 | Pressure drop in the recirculation line | $\Delta P$ | 2500 | Pa |
| 15 | Local pressure near the impeller | $P_{loc}$ | 120000 | Pa |
| 16 | Vapor pressure of a liquid | *Pv* | 12000 | Pa |
| 17 | Gravitational acceleration | *g* | 9.81 | m/s² |

The step-by-step selection, calculation, interpretation, and handling strategy are shown in **Table 54**. The relevant dimensionless numbers for the mixing vessel case are *Re*, *Fr*, *Eu*, *Np*, *Stk*, *We*, *Ca*, and $\sigma$c. The main engineering concerns are mixing regime, vortex formation, mixing power, particle suspension, droplet deformation, recirculation pressure drop, and cavitation near the impeller.

**Table 54.** Step-by-step selection, calculation, interpretation, and handling strategy for the mixing vessel case.

| STEP NO | QUESTION | ANSWER AND DECISION | HOW TO SOLVE | FORMULA AND CALCULATION | INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|---|
| 1 | Does the system have fluid flow? | Yes. Liquid circulates inside the mixing vessel. Use *Re*. | Calculate mixing *Re* in the impeller region. | $Re = 1100 \times 5.00 \times 0.30^2 / 0.020 = 24750$ | The mixing flow is turbulent around the impeller. | Maintain good circulation, but monitor excessive shear and energy use. |
| 2 | Does the system have a gas flow? | No. This case focuses on liquid or slurry mixing. *Ma* is not required. | No calculation is needed. | Not calculated. | Gas compressibility is not involved. | No *Ma*-based action is required. |
| 3 | Does gravity or free surface strongly affect the flow? | Yes. The mixing vessel has a free surface and possible vortex formation. Use *Fr*. | Calculate impeller-based *Fr*. | $Fr = 5.00^2 \times 0.30 / 9.81 = 0.765$ | Gravity still affects the free surface. Vortex may occur if the tank is not baffled. | Use baffles, adjust the liquid level, and avoid excessive impeller speed if a vortex is observed. |
| 4 | Does the system have a pressure drop? | Yes. Pressure drop occurs in the recirculation or discharge line. Use *Eu*. | Calculate *Eu* for the recirculation line. | $Eu = 2500 / (1100 \times 1.50^2) = 1.01$ | Pressure drop in the recirculation line is significant. | Check pump capacity, pipe diameter, valve opening, and possible fouling or blockage. |
| 5 | Does the system use an impeller or a mechanical mixer? | Yes. The vessel uses an impeller. Use *Np*. | Calculate *Np* for the mixing system. | $Np = 180 / (1100 \times 5.00^3 \times 0.30^5) = 0.539$ | Mixing power demand is moderate. | Check mixing homogeneity, solid suspension, baffle condition, and motor capacity. |

**Table 54 (continue).** Step-by-step selection, calculation, interpretation, and handling strategy for the mixing vessel case.

| STEP NO | QUESTION | ANSWER AND DECISION | HOW TO SOLVE | FORMULA AND CALCULATION | INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|---|
| 6 | Does the fluid carry particles, solids, or fouling material? | Yes. The slurry contains solid particles. Use *Stk*. | Calculate $\tau p$, $\tau f$, then *Stk*. | $\tau p = 2650 \times (1.0 \times 10^{-4})^2 / (18 \times 0.020) = 7.36 \times 10^{-5}$ s;<br><br>$\tau f = 0.30 / 4.71 = 0.0637$ s;<br><br>$Stk = 7.36 \times 10^{-5} / 0.0637 = 0.00116$ | Fine particles can follow the liquid motion easily. | Monitor bottom deposition, dead zones, and particle-size distribution. |
| 7 | Are droplets, bubbles, sprays, or jets formed or broken? | Yes. Droplet or emulsion breakup may occur near the impeller. Use *We*. | Calculate *We* for droplet deformation near the impeller. | $We = 1100 \times 4.71^2 \times 5.0 \times 10^{-4} / 0.050 = 244.0$ | Droplet deformation and breakup are likely near the impeller. | Control impeller speed, droplet size, and emulsifier use if droplet-size distribution is important. |
| 8 | Is viscous interfacial deformation important? | Yes. Droplet deformation may also be affected by viscosity. Use *Ca*. | Calculate *Ca* for viscous interfacial deformation. | $Ca = 0.020 \times 4.71 / 0.050 = 1.88$ | Viscous force significantly affects the interface. | Control viscosity, temperature, mixing time, and feed composition. |
| 9 | Can liquid vapor bubbles form due to low pressure? | Yes. Local low pressure may occur near the high-speed impeller region. Use $\sigma c$. | Calculate $\sigma c$ near the impeller. | $\sigma c = (120000 - 12000) / (0.5 \times 1100 \times 4.71^2) = 8.85$ | Cavitation risk is relatively low under the assumed condition. | Avoid excessive impeller speed, maintain liquid level, and prevent low-pressure operation. |

### 3.2.4. Integrated Case Study: Wastewater Aeration Basin

A wastewater aeration basin is used to mix wastewater, suspend sludge flocs, and supply oxygen through aeration. The system may include inlet flow, mechanical mixing, air diffusion, sludge suspension, diffuser pressure drop, and pump operation. The operating data are presented in **Table 55**.

**Table 55.** Operating data for the wastewater aeration basin case.

| NO | PARAMETER | SYMBOL | VALUE | UNIT |
|---|---|---|---|---|
| 1 | Wastewater density | $\rho l$ | 1000 | kg/m³ |
| 2 | Wastewater viscosity | $\mu f$ | $1.0 \times 10^{-3}$ | Pa.s |
| 3 | Surface tension | *sigma* | 0.072 | N/m |
| 4 | Inlet channel velocity | *vl* | 0.50 | m/s |
| 5 | Hydraulic depth | *Lc* | 0.50 | m |
| 6 | Pipe or channel diameter | *Dp* | 0.30 | m |
| 7 | Mechanical mixer power input | *Pw* | 500 | W |
| 8 | Impeller rotational speed | *N* | 3.00 | 1/s |
| 9 | Impeller diameter | $D_{imp}$ | 0.50 | m |
| 10 | Characteristic circulation velocity | *vc* | 0.80 | m/s |
| 11 | Bubble diameter | *db* | $3.0 \times 10^{-3}$ | m |
| 12 | Bubble relative velocity | *vb* | 0.25 | m/s |
| 13 | Sludge floc density | $\rho p$ | 1050 | kg/m³ |
| 14 | Sludge floc diameter | *Dp* | $2.0 \times 10^{-4}$ | m |
| 15 | Pressure drop through the diffuser | $\Delta P$ | 6000 | Pa |
| 16 | Air density | $\rho g$ | 1.2 | kg/m³ |
| 17 | Air velocity in the diffuser line | *vg* | 20.00 | m/s |
| 18 | Local pressure at the pump suction | $P_{loc}$ | 90000 | Pa |
| 19 | Vapor pressure of water | *Pv* | 3000 | Pa |
| 20 | Pump-line liquid velocity | *vp* | 4.00 | m/s |
| 21 | Gravitational acceleration | *g* | 9.81 | m/s² |

The step-by-step selection, calculation, interpretation, and handling strategy are shown in **Table 56**. The relevant dimensionless numbers for the wastewater aeration basin case are *Re*, *Fr*, *Eu*, *Np*, *Stk*, *We*, *Ca*, and $\sigma c$. The main engineering concerns are hydraulic flow, mixing power, sludge suspension, bubble deformation, diffuser pressure drop, and pump cavitation.

**Table 56.** Step-by-step selection, calculation, interpretation, and handling strategy for the wastewater aeration basin case.

| STEP NO | QUESTION | ANSWER AND DECISION | HOW TO SOLVE | FORMULA AND CALCULATION | INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|---|
| 1 | Does the system have fluid flow? | Yes. Wastewater flows through the inlet channel and basin. Use *Re*. | Calculate *Re* for the inlet wastewater flow. | $Re = 1000 \times 0.50 \times 0.30 / 1.0 \times 10^{-3} = 150000$ | Inlet wastewater flow is turbulent. | Use inlet structures or baffles to distribute flow and prevent short-circuiting. |
| 2 | Does the system have a gas flow? | No. Gas compressibility is not the main focus in this basin calculation. *Ma* is not required. | No calculation is needed. | Not calculated. | Gas compressibility is not dominant in the basin analysis. | No *Ma*-based action is required. |
| 3 | Does gravity or free surface strongly affect the flow? | Yes. The basin has open-channel and free-surface hydraulic behavior. Use *Fr*. | Calculate *Fr* for inlet channel flow. | $Fr = 0.50 / \text{sqrt}(9.81 \times 0.50) = 0.226$ | Gravity dominates over inertia. The flow is subcritical and relatively stable. | Maintain proper water depth and avoid very low velocities that may cause sedimentation. |
| 4 | Does the system have a pressure drop? | Yes. Pressure drop occurs through the air diffuser line. Use *Eu*. | Calculate *Eu* for the diffuser air line. | $Eu = 6000 / (1.2 \times 20.00^2) = 12.5$ | Diffuser pressure drop is significant. | Check diffuser fouling, blower capacity, pipe losses, and air distribution. |
| 5 | Does the system use an impeller or a mechanical mixer? | Yes. The basin uses a mechanical mixer. Use *Np*. | Calculate *Np* for the mechanical mixer. | $Np = 500 / (1000 \times 3.00^3 \times 0.50^5) = 0.593$ | Mixing power demand is moderate. | Check sludge suspension, dead zones, dissolved oxygen distribution, and mixer position. |

**Table 56 (continue).** Step-by-step selection, calculation, interpretation, and handling strategy for the wastewater aeration basin case.

| STEP NO | QUESTION | ANSWER AND DECISION | HOW TO SOLVE | FORMULA AND CALCULATION | INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|---|
| 6 | Does the fluid carry particles, solids, or fouling material? | Yes. Wastewater contains sludge flocs. Use *Stk*. | Calculate $\tau p$, $\tau f$, then *Stk*. | $\tau p$ = 1050 × (2.0 × $10^{-4}$)$^2$ / (18 × 1.0 × $10^{-3}$) = 2.33 × $10^{-3}$ s;<br><br>$\tau f$ = 0.50 / 0.80 = 0.625 s;<br><br>*Stk* = 2.33 × $10^{-3}$ / 0.625 = 0.00373 | Sludge flocs generally follow wastewater circulation. However, settling may still occur in low-velocity zones. | Monitor sludge accumulation, improve flow distribution, and avoid dead zones. |
| 7 | Are droplets, bubbles, sprays, or jets formed or broken? | Yes. Air bubbles are formed during aeration. Use *We*. | Calculate *We* for bubble deformation. | *We* = 1000 × 0.25$^2$ × 3.0 × $10^{-3}$ / 0.072 = 2.60 | Bubble deformation may occur. | Optimize diffuser type, air flow rate, and water depth to improve oxygen transfer. |
| 8 | Is viscous interfacial deformation important? | Yes. Bubble interface behavior may be affected by viscosity. Use *Ca*. | Calculate *Ca* for bubble-interface deformation. | *Ca* = 1.0 × $10^{-3}$ × 0.25 / 0.072 = 0.00347 | Surface tension dominates over the viscous force. | Consider wastewater composition, surfactants, and foam control if bubble stability changes. |
| 9 | Can liquid vapor bubbles form due to low pressure? | Yes. The wastewater system may use pumps or recirculation lines. Use σc. | Calculate σc near the pump suction. | σc = (90000 - 3000) / (0.5 × 1000 × 4.00$^2$) = 10.9 | Cavitation risk is relatively low under the assumed condition. | Maintain pump suction pressure, clean the inlet screen, and avoid excessive pump speed. |

### 3.2.5. Integrated Case Study: Aerated Fermentation Bioreactor

An aerated fermentation bioreactor is used to grow microorganisms in liquid broth while oxygen is supplied through sparging and distributed by mechanical agitation. The operating data are presented in **Table 57**.

**Table 57.** Operating data for the aerated fermentation bioreactor case.

| NO | PARAMETER | SYMBOL | VALUE | UNIT |
|---|---|---|---|---|
| 1 | Broth density | $\rho m$ | 1030 | kg/m$^3$ |
| 2 | Broth viscosity | $\mu f$ | 0.005 | Pa.s |
| 3 | Surface tension | *sigma* | 0.060 | N/m |
| 4 | Bioreactor diameter | *Dt* | 1.20 | m |
| 5 | Liquid height | *H* | 1.50 | m |
| 6 | Impeller diameter | $D_{imp}$ | 0.40 | m |
| 7 | Impeller rotational speed | *N* | 4.00 | 1/s |
| 8 | Impeller tip velocity | *vtip* | 5.03 | m/s |
| 9 | Characteristic circulation velocity | *vc* | 1.20 | m/s |
| 10 | Mixing power input | *Pw* | 900 | W |
| 11 | Cell or floc density | $\rho p$ | 1080 | kg/m$^3$ |
| 12 | Cell aggregate or floc diameter | *Dp* | $1.0 \times 10^{-4}$ | m |
| 13 | Bubble diameter | *db* | $3.0 \times 10^{-3}$ | m |
| 14 | Bubble relative velocity | *vb* | 0.30 | m/s |
| 15 | Air density | $\rho g$ | 1.2 | kg/m$^3$ |
| 16 | Air velocity in the sparger line | *vg* | 25.00 | m/s |
| 17 | Speed of sound in air | *c* | 346 | m/s |
| 18 | Pressure drop through sparger | $\Delta P$ | 8000 | Pa |
| 19 | Local pressure near the impeller | $P_{loc}$ | 120000 | Pa |
| 20 | Vapor pressure of broth | *Pv* | 3000 | Pa |
| 21 | Gravitational acceleration | *g* | 9.81 | m/s$^2$ |

The step-by-step selection, calculation, interpretation, and handling strategy are shown in **Table 58**. The relevant dimensionless numbers for the aerated fermentation bioreactor case are *Re*, *Ma*, *Fr*, *Eu*, *Np*, *Stk*, *We*, *Ca*, and $\sigma$c. The main engineering concerns are mixing regime, oxygen distribution, vortex control, biological suspension, bubble deformation, sparger pressure drop, gas compressibility, and cavitation near the impeller.

**Table 58.** Step-by-step selection, calculation, interpretation, and handling strategy for the aerated fermentation bioreactor case.

| STEP NO | QUESTION | ANSWER AND DECISION | HOW TO SOLVE | FORMULA AND CALCULATION | INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|---|
| 1 | Does the system have fluid flow? | Yes. Broth circulates inside the bioreactor. Use *Re*. | Calculate mixing *Re* in the impeller region. | $Re = 1030 \times 4.00 \times 0.40^2 / 0.005 = 131840$ | Mixing is turbulent around the impeller. | Maintain good circulation, but monitor shear-sensitive cells. |
| 2 | Does the system have a gas flow? | Yes. Air flows through the sparger line. Use *Ma*. | Calculate *Ma* for air flow. | $Ma = 25.00 / 346 = 0.072$ | Gas compressibility is not dominant because $Ma < 0.3$. | Nearly incompressible gas-flow assumption may be acceptable for preliminary air-line analysis. |
| 3 | Does gravity or free surface strongly affect the flow? | Yes. The bioreactor has a free surface and possible vortex formation. Use *Fr*. | Calculate impeller-based *Fr*. | $Fr = 4.00^2 \times 0.40 / 9.81 = 0.652$ | Gravity still affects free-surface behavior. | Use baffles to reduce vortex and improve axial and radial circulation. |
| 4 | Does the system have a pressure drop? | Yes. Pressure drop occurs through the sparger line. Use *Eu*. | Calculate *Eu* for the sparger air line. | $Eu = 8000 / (1.2 \times 25.00^2) = 10.7$ | Sparger pressure drop is significant. | Check blower capacity, sparger clogging, and air-line pressure drop. |
| 5 | Does the system use an impeller or a mechanical mixer? | Yes. The bioreactor uses an impeller. Use *Np*. | Calculate *Np* for the mixing system. | $Np = 900 / (1030 \times 4.00^3 \times 0.40^5) = 1.33$ | Mixing power demand is moderate to high. | Check motor capacity, oxygen transfer, and possible shear damage. |
| 6 | Does the fluid carry particles, solids, or biological flocs? | Yes. The broth contains cells or biological flocs. Use *Stk*. | Calculate $\tau p$, $\tau f$, then *Stk*. | $\tau p = 1080 \times (1.0 \times 10^{-4})^2 / (18 \times 0.005) = 1.20 \times 10^{-4}$ s;<br><br>$\tau f = 0.40 / 5.03 = 0.0795$ s;<br><br>$Stk = 1.20 \times 10^{-4} / 0.0795 = 0.00151$ | Small cell aggregates generally follow broth circulation. | Monitor large pellets, dense flocs, and low-circulation zones. |

**Table 58 (continue).** Step-by-step selection, calculation, interpretation, and handling strategy for the aerated fermentation bioreactor case.

| STEP NO | QUESTION | ANSWER AND DECISION | HOW TO SOLVE | FORMULA AND CALCULATION | INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|---|
| 7 | Are droplets, bubbles, sprays, or jets formed or broken? | Yes. Air bubbles are formed and dispersed in the broth. Use *We*. | Calculate *We* for bubble deformation. | $We = 1030 \times 0.30^2 \times 3.0 \times 10^{-3} / 0.060 = 4.64$ | Bubble deformation may occur. | Optimize sparger design, gas flow rate, and impeller speed for oxygen transfer. |
| 8 | Is viscous interfacial deformation important? | Yes. Bubble interface behavior may be affected by broth viscosity. Use *Ca*. | Calculate *Ca* for bubble-interface deformation. | $Ca = 0.005 \times 0.30 / 0.060 = 0.0250$ | Surface tension dominates over the viscous force. | Consider medium composition, antifoam use, and foam control if bubble stability changes. |
| 9 | Can liquid vapor bubbles form due to low pressure? | Yes. Local low pressure may occur near the high-speed impeller. Use $\sigma c$. | Calculate $\sigma c$ near the impeller region. | $\sigma c = (120000 - 3000) / (0.5 \times 1030 \times 5.03^2) = 8.98$ | Cavitation risk is relatively low under the assumed condition. | Avoid excessive impeller speed, maintain adequate liquid level, and prevent low-pressure operation. |

### 3.2.6. Integrated Case Study: Spray Drying Process

A spray drying process is used to convert liquid feed into dry powder using hot drying air. The system includes feed preparation, feed pipe, atomizer, drying chamber, wall region, cyclone, and exhaust duct. The operating data are presented in **Table 59**.

**Table 59.** Operating data for the spray drying case.

| NO | PARAMETER | SYMBOL | VALUE | UNIT |
|---|---|---|---|---|
| 1 | Feed density | $\rho m$ | 1100 | kg/m³ |
| 2 | Feed viscosity | $\mu f$ | 0.020 | Pa.s |
| 3 | Feed surface tension | *sigma* | 0.050 | N/m |
| 4 | Feed pipe diameter | *Dp* | 0.020 | m |
| 5 | Feed velocity in the pipe | *vl* | 1.50 | m/s |
| 6 | Feed pressure before the nozzle | $P_{loc}$ | 300000 | Pa |
| 7 | Vapor pressure of feed | *Pv* | 12000 | Pa |
| 8 | Nozzle or atomizer velocity | *vn* | 20.00 | m/s |
| 9 | Droplet diameter | *db* | $1.0 \times 10^{-4}$ | m |
| 10 | Drying-air density | $\rho g$ | 1.0 | kg/m³ |
| 11 | Drying-air viscosity | $\mu g$ | $2.0 \times 10^{-5}$ | Pa.s |
| 12 | Drying-air velocity | *vg* | 30.00 | m/s |
| 13 | Speed of sound in hot air | *c* | 412 | m/s |
| 14 | Characteristic chamber length | *Lc* | 1.00 | m |
| 15 | Pressure drop through the air distributor | $\Delta P$ | 1000 | Pa |
| 16 | Particle density after drying | $\rho p$ | 1200 | kg/m³ |
| 17 | Particle diameter | *Dp* | $5.0 \times 10^{-5}$ | m |
| 18 | Mixing power input | *Pw* | 150 | W |
| 19 | Impeller speed in the feed tank | *N* | 5.00 | 1/s |
| 20 | Impeller diameter | $D_{imp}$ | 0.30 | m |
| 21 | Gravitational acceleration | *g* | 9.81 | m/s² |

The step-by-step selection, calculation, interpretation, and handling strategy are shown in **Table 60**. The relevant dimensionless numbers for the spray drying case are *Re*, *Ma*, *Fr*, *Eu*, *Np*, *Stk*, *We*, *Ca*, and $\sigma c$. The main engineering concerns are feed flow regime, gas compressibility, droplet transport, pressure drop, feed mixing, particle response, atomization, feed viscosity, and cavitation near the nozzle.

**Table 60.** Step-by-step selection, calculation, interpretation, and handling strategy for the spray drying case.

| STEP NO | QUESTION | ANSWER AND DECISION | HOW TO SOLVE | FORMULA AND CALCULATION | INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|---|
| 1 | Does the system have fluid flow? | Yes. Liquid feed flows in the pipe. Use *Re*. | Calculate *Re* for feed pipe flow. | $Re$ = 1100 × 1.50 × 0.020 / 0.020 = 1650 | Feed flow is laminar to transitional. | Monitor pressura e drop, feed stability, and possible flow non-uniformity. |
| 2 | Does the system have gas flow? | Yes. Hot drying air flows through the chamber. Use *Ma*. | Calculate *Ma* for drying-air flow. | $Ma$ = 30.00 / 412 = 0.073 | Gas compressibility is not dominant because $Ma < 0.3$. | Nearly incompressible gas-flow assumption may be acceptable for preliminary analysis. |
| 3 | Does gravity or free surface strongly affect the flow? | Yes. Droplet transport in the chamber is affected by gas momentum and gravity. Use *Fr*. | Calculate *Fr* for the drying chamber. | $Fr$ = 30.00 / sqrt(9.81 × 1.00) = 9.58 | Gas inertia dominates over gravity. | Optimize air distributor, spray angle, and chamber geometry to reduce wall deposition. |
| 4 | Does the system have a pressure drop? | Yes. Pressure drop occurs through the air distributor. Use *Eu*. | Calculate *Eu* for drying-air flow. | $Eu = 1000 / (1.0 \times 30.00^2) = 1.11$ | Pressure drop through the air distributor is significant. | Check blower capacity, air distributor design, and possible fouling or blockage. |
| 5 | Does the system use an impeller or a mechanical mixer? | Yes. The feed tank uses mixing before atomization. Use *Np*. | Calculate *Np* for the feed tank. | $Np = 150 / (1100 \times 5.00^3 \times 0.30^5) = 0.449$ | Mixing power demand is moderate. | Check feed homogeneity, solid suspension, viscosity, and atomizer feed stability. |

**Table 60.** Step-by-step selection, calculation, interpretation, and handling strategy for the spray drying case.

| STEP NO | QUESTION | ANSWER AND DECISION | HOW TO SOLVE | FORMULA AND CALCULATION | INTERPRETATION | HANDLING STRATEGY |
|---|---|---|---|---|---|---|
| 6 | Does the fluid carry particles, solids, or droplets? | Yes. Droplets and dried particles are transported by drying air. Use *Stk*. | Calculate $\tau p$, $\tau f$, then *Stk*. | $\tau p = 1200 \times (5.0 \times 10^{-5})^2 / (18 \times 2.0 \times 10^{-5}) = 0.00833$ s;<br><br>$\tau f = 1.00 / 30.00 = 0.0333$ s;<br><br>$Stk = 0.00833 / 0.0333 = 0.250$ | Particles mostly follow gas flow, but particle inertia is not negligible. | Monitor wall deposition, cyclone inlet behavior, and particle residence time. |
| 7 | Are droplets, bubbles, sprays, or jets formed or broken? | Yes. Liquid feed is atomized into droplets. Use *We*. | Calculate *We* near the atomizer. | $We = 1.0 \times 20.00^2 \times 1.0 \times 10^{-4} / 0.050 = 0.800$ | Droplet deformation may begin, but atomization may be limited. | Increase atomizing velocity, adjust nozzle design, or reduce surface tension if suitable. |
| 8 | Is viscous interfacial deformation important? | Yes. Feed viscosity affects droplet deformation and atomization. Use *Ca*. | Calculate *Ca* for feed droplet deformation. | $Ca = 0.020 \times 20.00 / 0.050 = 8.00$ | Viscous effect is strong. | Reduce feed viscosity, increase feed temperature if allowed, control solid concentration, and improve atomizer energy. |
| 9 | Can liquid vapor bubbles form due to low pressure? | Yes. Local low pressure may occur near the nozzle or the feed restriction. Use $\sigma c$. | Calculate $\sigma c$ near the nozzle. | $\sigma c = (300000 - 12000) / (0.5 \times 1100 \times 20.00^2) = 1.31$ | Cavitation risk should be checked. | Increase local pressure, reduce restriction, reduce velocity, and monitor feed temperature. |

## 4. CONCLUSION

This paper explained the calculation and interpretation of selected dimensionless numbers in fluid mechanics (*Re*, *Ma*, *Fr*, *Eu*, *Np*, *Stk*, *We*, *Ca*, and $\sigma c$) for chemical engineering applications. Each number represents a specific physical phenomenon and can support process analysis. Integrated case examples confirmed that real equipment involves multiple fluid-flow effects. Dimensionless-number analysis is useful for design, scale-up, troubleshooting, operation, and optimization.

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## 6. AUTHORS' NOTE

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